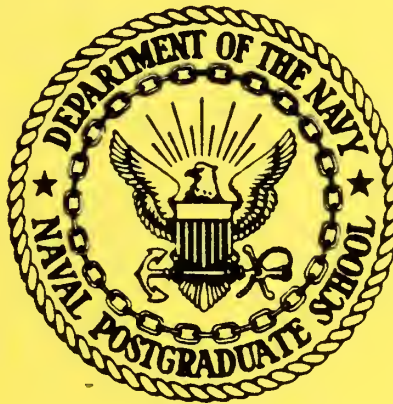


NPS-69-81-005PR

NAVAL POSTGRADUATE SCHOOL

Monterey, California



STRENGTH ANALYSIS OF COMPOSITE PLATES
IN A THERMAL ENVIRONMENT

by

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September 1981

Progress Report for Period December 1980
- September 1981

Approved for Public Release: Distribution Unlimited

Prepared for: Naval Weapons Center
China Lake, CA 93555

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS69-81-005PR	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) STRENGTH ANALYSIS OF COMPOSITE PLATES IN A THERMAL ENVIRONMENT		5. TYPE OF REPORT & PERIOD COVERED Progress December 80 - September 81
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) David Salinas		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 63514N;W0592001 N60530 81WR30111
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Weapons Center China Lake, CA 93555		12. REPORT DATE September 1981
		13. NUMBER OF PAGES 51
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for Public Release: Distribution Unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Composites, thermal environment, strength analysis		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) An investigation of the strength of a fiber reinforced composite plate subjected to external loads and thermal environment is considered. The analysis is restricted to symmetric layups. The general Tsai-Wu failure criterion is used to predict failure. The analyses has resulted in a computer program which will predict the loads at which a composite plate will fail under a specified thermal environment.		

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NOTATION

a, b, c, \dots	Eq.(64) coefficients, defined by Eq's.(65)-(67)
\tilde{A}	laminated inplane stiffness matrix; Eq.(34)
\tilde{B}	laminated coupling stiffness matrix; Eq.(35)
\tilde{C}_i	lamina stiffness matrix; Eq.(1)
\tilde{D}	laminated moment stiffness matrix; Eq.(36)
\tilde{e}	strain
E_x, E_y	lamina longitudinal and transverse stiffnesses
$F_{xx}, F_x, F_{yy}, F_y, F_{xy}, F_{ss}$	Tsai-Wu failure parameters; Eq's.(56) and (58)
G_{xy}	lamina shear modulus
\tilde{M}	laminated moment vector; Eq.(26)
\tilde{M}_T	laminated thermal moment vector; Eq.(38)
\tilde{N}	laminated force vector; Eq.(25)
\tilde{N}_T	laminated thermal force vector; Eq.(37)
\tilde{Q}_i	lamina stiffness matrix; Eq.(13)
R_i	lamina strength ratio; Eq.(60)
\tilde{S}_i	lamina compliance matrix; Eq.(13)
t	plate thickness
T_i	lamina temperature
u, v, w	displacements along the x, y, z axes
x_i, y_i, z_i	lamina axes
$\bar{x}, \bar{y}, \bar{z}$	laminated axes
α	vector of thermal coefficients of expansion
β	vector of thermal coefficients
γ	engineering shear strain
κ	laminated curvature vector
ϕ_i	orientation angle of i-th ply
θ_i	temperature of i-th ply; Eq.(2)
ν_{xy}, ν_{yx}	longitudinal and transverse Poisson's ratio
X, X', Y, Y', S	lamina strengths

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1.0 Introduction

In a severe thermal environment, materials generally experience a degradation of stiffness and strength, and as a result there is a diminishment in the load carrying capacity of the material. In order to design a structure which will perform safely in a thermal environment, the structural engineer must have an analysis program which will predict the behavior of the structure. This report documents an analysis for the behavior of a fiber reinforced composite plate subjected to loads in a thermal environment.

2.0 Description of Fiber Reinforced Composite Plates

As a result of imperfections, such as dislocations and surface scratches, the strength of bulk material is orders of magnitude less than the strength of the material in crystal or whisker form. The cost of producing crystals is so great as to prohibit production except on a limited basis. On the other hand, fibers of a material can be produced at a cost which permits their application to structural systems.

Depending upon it's diameter, a fiber of a material has a strength intermediate to the bulk and crystal strengths of the material. For example, the strength of steel fiber is approximately 4.14 MPa (600,000 psi), compared to a strength of 1.04 MPa (150,000 psi) for steel in it's common bulk form. Thus the use of steel fiber in place of common steel results in a four-fold reduction in weight. Actually the use of fiber requires their embedment in a matrix of another material in order to achieve an useful structural form. However even with embedment, fiber reinforced materials offer a significant increase in the strength to weight ratio over common materials. This is the primary reason for using composite materials.

One of the most common useful structural component is the plate element. A fiber reinforced composite plate (or simply, a laminate) consists of layers of lamina (or plies)

stacked one upon another. A lamina itself is comprised of unidirectional fibers embedded in a binding material, called the matrix. Generally, the binding matrix material is a much weaker and more ductile material than the fiber reinforcement embedded within it. Some common fiber/matrix combinations are graphite/epoxy, glass/epoxy, and boron/aluminum. Some idea of the dimensions involved can be obtained from the dimensions of a typical graphite/epoxy lamina;

- i) fiber diameter is approximately .008 mm. (.0003 in.)
- ii) lamina thickness is approximately .125 mm. (.005 in.)
- iii) fiber volume to total volume (called the fiber fraction) is about 0.5

For these dimensions, a 12.5 mm. (0.5 in.) thick laminate will consist of 100 plies (lamina) stacked one upon another.

As a result of unidirectional fibers embedded within it, a lamina is not isotropic. The stiffness of a lamina along its fiber direction (called the longitudinal stiffness) is much greater than the stiffness of the lamina in the direction orthogonal to the fiber (called the transverse stiffness). For example, in the case of a typical graphite/epoxy lamina, the longitudinal stiffness (E_x or E_L) and the transverse stiffness (E_y or E_T) are 181 GPa (26×10^6 psi) and 10.3 GPa (1.5×10^6 psi), respectively.

Stacking lamina of different orientation and thickness upon one another to form a composite plate results in an orthotropic body. One of the most attractive features of a composite plate is the capacity for design of material properties, an option not previously available to structural engineers.

3.0 Lamina and Laminate Coordinate Systems

In dealing with fiber reinforced composite plates, it is necessary to employ two types of coordinate systems. The first type of coordinate system is associated with a lamina. This coordinate system is referred to as the natural coordinate

system, or the local coordinate system, or the lamina (or ply) coordinate system. Here we assign the cartesian pair of x, y axes to the plane of the lamina, with the x -axis along the fiber direction of the lamina. Since adjacent lamina have different fiber orientation with respect to one another, there are as many natural coordinate systems as there are plies. The i -th lamina (ply) has the coordinate system $(x, y)_i$ or (x_i, y_i) . This is shown in Fig.1 below.

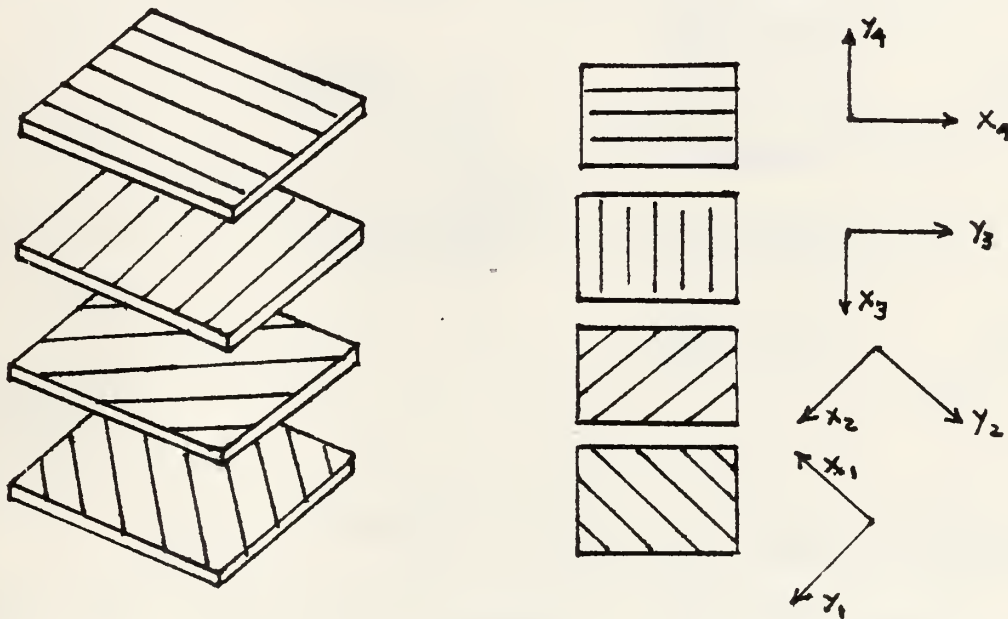


Figure 1. Lamina Layup to Form a Laminate

The second type of coordinate system associated with a composite plate is the coordinate system which ties all of the lamina coordinate systems to a single pair of axes, called the laminate or global coordinate system, and denoted by (\bar{x}, \bar{y}) . These axes lie in a plane parallel to the lamina planes, and are generally located at the midplane of the laminate, as shown in Fig.2 for a 3 ply laminate.

Each of the local (lamina) coordinate systems are related to the global (laminate) coordinate system by the angle ϕ_i , where ϕ_i is the angle between the x_i axis of the i -th

lamina and the \bar{x} axis of the laminate. In this way, the orientation layup of a n-ply laminate can be denoted by the notation $[\phi_1, \phi_2, \phi_3, \dots, \phi_n]$. There is no requirement that

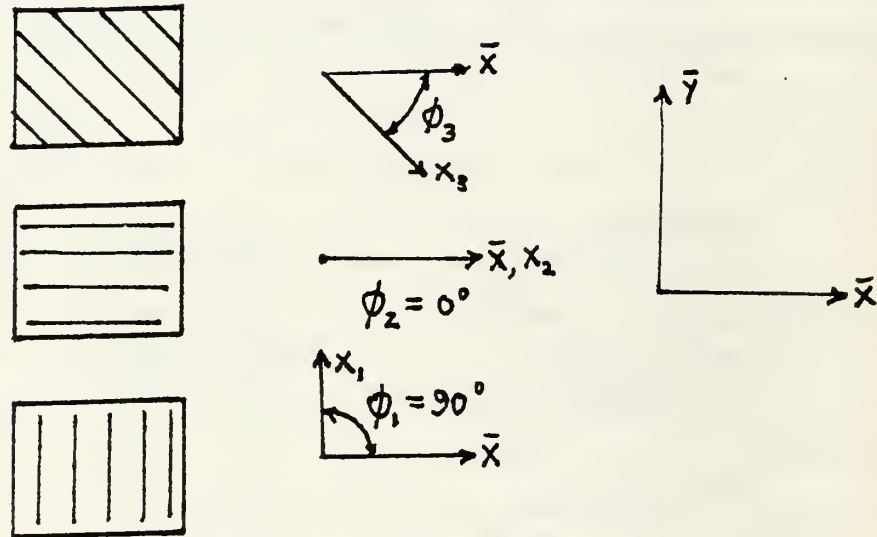


Figure 2. Lamina and Laminate Coordinate Systems

the lamina be of equal thickness, although this is a common construction.

4.0 Stress-Strain Relations in Natural Coordinates for a Lamina in Plane Stress

In the case of plates subjected to inplane and transverse loads, the result is the plane stress state. This means that for a lamina in the x, y plane, interlaminar stresses σ_{zz} , σ_{zy} , and σ_{zx} are negligible compared to inplane stresses σ_{xx} , σ_{yy} , and σ_{xy} , and are therefore ignored. It should be noted in passing that interlaminar stresses can be very large in local regions of boundaries. (See Ref.1 for details). In this report interlaminar stresses are not considered.

In terms of it's own natural coordinate system, the stress-strain relations for the i -th lamina, in tensor notation, are

$$(\sigma_{jk})_i = (C_{jkr s} e_{rs} - \beta_{jk} \theta)_i \quad j, k, r, s = x_i, y_i \quad (1)$$

where σ_{jk} and e_{rs} are the second order stress and strain tensors, β_{jk} is the second order tensor of thermal coefficients, $C_{jkr s}$ is the fourth order tensor of stiffness coefficients, and θ is the difference in temperature from the stress free temperature. That is, if T is the stress free (curing) temperature, and T_i is the temperature of the i -th lamina, then

$$\theta_i = (T_i - T) \quad (2)$$

Throughout this report, the Einstein summation convention is in effect for all tensor equations.

Alternatively, Eq.(1) may be written for strains in terms of stress, as

$$(e_{jk})_i = (S_{jkr s} \sigma_{rs} + \alpha_{jk} \theta)_i \quad j, k, r, s = x_i, y_i \quad (3)$$

where $S_{jkr s}$ is the fourth order tensor of compliance coefficients, and α_{jk} is the tensor of thermal expansion coefficients. We note that properties of a lamina, such as stiffnesses, compliances, thermal coefficients, and thermal expansion coefficients, are functions of temperature. However over narrow ranges of temperature, properties can be treated as constant without introducing significant error into the analysis.

As a matter of convenience, hereafter we proceed using contracted "vector" notation. It should always be kept in mind that stresses, strains, thermal coefficients, thermal expansion coefficients, stiffnesses, and compliances are, in fact, tensors, not vectors, and therefore obey the appropriate tensor transformation laws when transformations occur. The second order tensors (σ_{ij} , e_{ij} , β_{ij} , and α_{ij}) in the contracted notation have a single subscript in accordance with Table 1, and the fourth order tensors (S_{ijkl} and C_{ijkl}) carry the double subscript notation shown in Table 2.

		Tensor Subscripts	Contracted Notation
		xxxx	11
		xyyy	12
		xxxy and xxyx	16
		yyxx	21
		yyyy	22
		yyxy and yyyx	26
		xyxx and yxxx	61
		xyyy and yxyy	62
		xyxy and yxyx	66

Tensor Subscripts	Contracted Notation
xx	1
yy	2
xy and yx	6

Table 1. Contracted Notation for 2nd order Tensors

Table 2. Contracted Notation for 4th order Tensors

For example, using the symmetry conditions, $e_{xy} = e_{yx}$, $C_{xxxy} = C_{xxyx}$ and so on, and defining the engineering shear strain $\gamma_{xy} = (e_{xy} + e_{yx})$, equations (1) become

$$\begin{aligned}
 [\sigma_1 &= C_{11}e_1 + C_{12}e_2 + C_{16}\gamma_6 - \beta_1\theta]_i \\
 [\sigma_2 &= C_{21}e_1 + C_{22}e_2 + C_{26}\gamma_6 - \beta_2\theta]_i \\
 [\sigma_6 &= C_{61}e_1 + C_{62}e_2 + C_{66}\gamma_6 - \beta_6\theta]_i
 \end{aligned} \tag{4}$$

or, in vector notation

$$\tilde{\sigma}_i = (\tilde{C}\tilde{e} - \beta\theta)_i \tag{5}$$

where the i subscript denotes the equation is for the i -th lamina. In equation (5), and hereafter, a single tilde under a quantity denotes the quantity is treated as a 3x1 vector, a double tilde under a quantity means the quantity is treated as a 3x3 matrix, and a quantity without a tilde beneath it denotes that the quantity is a scalar.

As a result of the natural coordinates being along and orthogonal to the fibers of a lamina, the natural coordinates are the principal axes of the lamina. Thus the normal and shear effects uncouple, and Eq. (5) takes on the special form,

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix}_i = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} e_1 \\ e_2 \\ \gamma_6 \end{Bmatrix}_i - \theta_i \begin{Bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{Bmatrix}_i \quad (6)$$

That is, in the natural coordinates of a lamina, $C_{16}=C_{26}=C_{66}=0$, and $\beta_6=0$. Furthermore, as the xy plane is a plane of symmetry, $C_{12}=C_{21}$, and therefore \tilde{C} is a symmetric matrix. Thus a lamina has four independent stiffness coefficients (C_{11}, C_{12}, C_{22} , and C_{66}), and two independent thermal coefficients (β_1 , and β_2). Such materials are called orthotropic materials. It is important to note that the special form of \tilde{C} in Eq.(6), associated with uncoupled normal and shear behavior, is valid only in natural coordinates of a lamina.

The notation used most frequently in the composites literature is \tilde{S} for compliance, and \tilde{Q} rather than \tilde{C} for stiffness. Hereafter we shall use \tilde{S} and \tilde{Q} . In contracted notation, Eq's.(1) and (3) are,

$$\tilde{\sigma}_i = (\tilde{Q}e - \tilde{\beta}\theta)_i \quad (7)$$

and

$$\tilde{e}_i = (\tilde{S}\tilde{\sigma} + \tilde{\alpha}\theta)_i \quad (8)$$

Premultiplication of Eq.(7) by \tilde{Q}^{-1} and solving for strain \tilde{e} gives

$$\tilde{e}_i = (\tilde{Q}^{-1}\tilde{\sigma} + \tilde{Q}^{-1}\tilde{\beta}\theta)_i \quad (9)$$

A comparison of equations (8) and (9) shows the relations between \tilde{Q} and \tilde{S} , and $\tilde{\beta}$ and $\tilde{\alpha}$ are

$$\tilde{Q} = \tilde{S}^{-1} \quad (10)$$

and

$$\tilde{\alpha} = \tilde{Q}^{-1}\tilde{\beta} \quad (11)$$

The stiffness coefficients Q_{jk} are related to the common engineering coefficients E_x, E_y, ν_{xy} , and G_{xy} , by Eq's.(12). The engineering coefficients are obtained by performing the simple tension and shear tests on a lamina as depicted in

Figure 3. Pulling a lamina along the fiber direction (Fig.3a) gives the longitudinal stiffness, E_x . The tension test in the orthogonal direction (Fig.3b) gives the transverse stiffness E_y . The longitudinal Poisson's ratio ν_{xy} is the ratio of e_x to e_y obtained in the longitudinal tension test. The longitudinal shear modulus G_{xy} is obtained from the shear test shown in Fig.3c. The relations between S_{jk} and engineering coefficients are,

$$\begin{aligned} S_{11} &= (1/E_x) \\ S_{12} &= S_{21} = -(\nu_{xy}/E_x) = -(\nu_{yx}/E_y) \\ S_{22} &= (1/E_y) \\ S_{66} &= (1/G_{xy}) \end{aligned} \quad (12)$$

Note that the second of these relations defines a transverse Poisson's ratio, ν_{yx} . The x and y subscripts denoting longitudinal and transverse properties are quite often replaced by T and L subscripts, i.e., E_L denotes the longitudinal stiffness of a lamina, E_T is the transverse stiffness, and so on. Using Eq.(10), the relations between stiffness coefficients Q_{ij} and the engineering constants are found to be,

$$\begin{aligned} Q_{11} &= (E_x / (1 - \nu_{xy}\nu_{yx})) \\ Q_{12} &= Q_{21} = (\nu_{xy}E_x / (1 - \nu_{xy}\nu_{yx})) = (\nu_{yx}E_y / (1 - \nu_{xy}\nu_{yx})) \\ Q_{22} &= (E_y / (1 - \nu_{xy}\nu_{yx})) \\ Q_{66} &= G_{xy} \end{aligned} \quad (13)$$

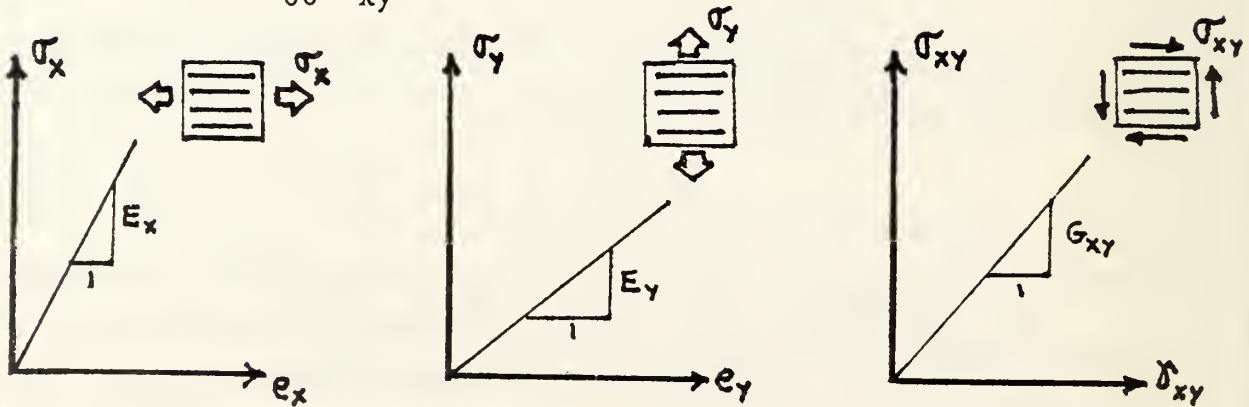


Figure 3. Experimental Tests for Engineering Stiffness Constants

5.0 Stress-Strain Relations for a Lamina in Plane Stress with Respect to Laminate Axes

In order to develop the equations for a laminate, it is necessary to relate each lamina comprising the laminate to a common coordinate system, the laminate axes. This is accomplished by coordinate transformation from lamina axes to laminate axes. Each lamina coordinate system $(x, y, z)_i$ is oriented to the laminate axes $(\bar{x}, \bar{y}, \bar{z})$, where z_i and \bar{z} coincide, by an angle ϕ_i . Recall from Fig.2, that ϕ_i is the angle between the x_i axis of the i -th lamina and the \bar{x} axis of the laminate.

Hereafter all quantities with a bar over the quantity symbol means that the quantity is with respect to laminate axes, and quantities without a bar over them means the quantities are with respect to lamina axes. Thus the stresses, strains, thermal coefficients, and stiffnesses with respect to the laminate coordinate system are $\bar{\sigma}, \bar{\epsilon}, \bar{\beta}$, and \bar{Q} . Using the tensor transformation laws for the second order tensors,

$$\begin{aligned}(\sigma_{jk})_i &= (l_{jm} l_{kn} \bar{\sigma}_{mn})_i \\(e_{jk})_i &= (l_{jm} l_{kn} \bar{e}_{mn})_i \\(\beta_{jk})_i &= (l_{jm} l_{kn} \bar{\beta}_{mn})_i\end{aligned}\tag{14}$$

where l_{rs} is the cosine of the angle between the r -th lamina axis and the s -th laminate axis, gives the expressions for lamina quantities with respect to laminate axes. Denoting the transformation between coordinate systems by the matrix T , Eq's.(14) may be written,

$$\begin{aligned}\bar{\sigma}_i &= (T \bar{\sigma})_i \\ \bar{\epsilon}_i &= (T \bar{\epsilon})_i \\ \bar{\beta}_i &= (T \bar{\beta})_i\end{aligned}\tag{15}$$

Substitution of Eq's.(15) into Eq's.(7) gives

$$\bar{\sigma}_i = Q_i T_i \bar{\epsilon}_i - T_i \bar{\beta}_i \theta_i$$

Premultiplication through by the inverse of T_i gives,

$$\bar{\sigma}_i = T_i^{-1} Q_i T_i \bar{\epsilon}_i - \bar{\beta}_i \theta_i$$

Denoting the matrix $T_i^{-1} Q_i T_i$ by \bar{Q}_i , where \bar{Q}_i is the 3x3 stiffness matrix of the i-th lamina with respect to laminate axes, we obtain the stress-strain relations for a lamina in terms of laminate axes as,

$$\bar{\sigma}_i = \bar{Q}_i \bar{\epsilon}_i - \bar{\beta}_i \theta_i$$

Recalling Eq.(11), yields the more useful expression (since the coefficients of thermal expansion are generally known),

$$\bar{\sigma}_i = \bar{Q}_i (\bar{\epsilon}_i - \bar{\alpha}_i \theta_i) \quad (16)$$

where the i subscript refers to the i-th lamina. Explicitly Eq's.(16) are,

$$\begin{bmatrix} \bar{\sigma}_1 \\ \bar{\sigma}_2 \\ \bar{\sigma}_6 \end{bmatrix}_i = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix}_i \begin{bmatrix} \bar{\epsilon}_1 \\ \bar{\epsilon}_2 \\ \gamma_6 \end{bmatrix}_i - \theta_i \begin{bmatrix} \bar{\alpha}_1 \\ \bar{\alpha}_2 \\ \bar{\alpha}_6 \end{bmatrix}_i \quad (17)$$

where, upon using $c_i = \cos \phi_i$ and $s_i = \sin \phi_i$, we obtain,

$$\begin{aligned} (\bar{Q}_{11})_i &= [Q_{11}c^4 + 2(Q_{12}+2Q_{66})s^2c^2 + Q_{22}s^4]_i \\ (\bar{Q}_{12})_i &= [(Q_{11}+Q_{22}-4Q_{66})s^2c^2 + Q_{12}(s^4+c^4)]_i \\ (\bar{Q}_{22})_i &= [Q_{11}s^4 + 2(Q_{12}+2Q_{66})s^2c^2 + Q_{22}c^4]_i \\ (\bar{Q}_{16})_i &= [(Q_{11}-Q_{12}-2Q_{66})sc^3 + (Q_{12}-Q_{22}+2Q_{66})s^3c]_i \\ (\bar{Q}_{26})_i &= [(Q_{11}-Q_{12}-2Q_{66})s^3c + (Q_{12}-Q_{22}+2Q_{66})sc^3]_i \\ (\bar{Q}_{66})_i &= [(Q_{11}+Q_{22}-2Q_{12}-2Q_{66})s^2c^2 + Q_{66}(s^4+c^4)]_i \end{aligned} \quad (18)$$

We note from Eq's.(18) that the stiffness matrix of a lamina with respect to the laminate coordinate system is a full 3×3 matrix. However, there are still only four independent coefficients; any two of the \bar{Q}_{jk} coefficients may be written as functions of the remaining four coefficients. The \bar{Q} matrix is symmetric, that is, $\bar{Q}_{12} = \bar{Q}_{21}$, $\bar{Q}_{16} = \bar{Q}_{61}$, and $\bar{Q}_{26} = \bar{Q}_{62}$.

The fact that \bar{Q} is, in general, full, means that coupling between normal and shear behavior occurs. In contrast, for isotropic materials, normal and shear behaviors uncouple. For cross ply laminates, that is laminates with orthogonal plies, subjected to inplane forces and bending moments, normal and shear behaviors uncouple. For angle ply laminates, that is laminates with some non-orthogonal plies, coupling between normal and shear behaviors occurs. This difference in behavior between cross ply laminates and angle ply laminates is shown by an illustrative example in section 11.

6.0 Strain-Displacement Relations for a Laminate

In accordance with common thin plate theory, the assumption that planes before loading remain planes after loading (Kirchhoff-Love theory) is taken as the basis of deformation of a laminate. In this case, the displacement field is given by,

$$\left. \begin{aligned} \bar{u} &= \bar{u}_0 - z(\partial \bar{w} / \partial \bar{x}) \\ \bar{v} &= \bar{v}_0 - z(\partial \bar{w} / \partial \bar{y}) \end{aligned} \right\} \quad (19)$$

where \bar{u} and \bar{v} are the displacements in the \bar{x} and \bar{y} directions, \bar{u}_0 and \bar{v}_0 are the midplane displacements, \bar{w} is the displacement in the \bar{z} direction, and \bar{z} is the distance from the midplane to the point of interest.

Using the linear strain-displacement relations,

$$\left. \begin{aligned} \bar{e}_x &= (\partial \bar{u} / \partial \bar{x}) & \bar{e}_y &= (\partial \bar{v} / \partial \bar{y}) \\ \bar{\gamma}_{xy} &= (\partial \bar{u} / \partial \bar{y}) + (\partial \bar{v} / \partial \bar{x}) \end{aligned} \right\} \quad (20)$$

equations (19) become,

$$\left. \begin{aligned} \bar{e}_1 &= (\partial \bar{u}_0 / \partial \bar{x}) - \bar{z} (\partial^2 \bar{w} / \partial \bar{x}^2) \\ \bar{e}_2 &= (\partial \bar{v}_0 / \partial \bar{y}) - \bar{z} (\partial^2 \bar{w} / \partial \bar{y}^2) \\ \bar{\gamma}_6 &= (\partial \bar{u}_0 / \partial \bar{y} + \partial \bar{v}_0 / \partial \bar{x}) - 2\bar{z} (\partial^2 \bar{w} / \partial \bar{x} \partial \bar{y}) \end{aligned} \right\} \quad (21)$$

Equations (21) may be written in vector form as,

$$\bar{\mathbf{e}} = \bar{\mathbf{e}}_0 + \bar{z} \bar{\mathbf{k}} \quad (22)$$

where $\bar{\mathbf{e}}_0$ is the strain vector associated with midplane extensional displacements given by

$$\left. \begin{aligned} (\bar{e}_1)_0 &= \partial \bar{u}_0 / \partial \bar{x} \\ (\bar{e}_2)_0 &= \partial \bar{v}_0 / \partial \bar{y} \\ (\bar{\gamma}_6)_0 &= \partial \bar{u}_0 / \partial \bar{y} + \partial \bar{v}_0 / \partial \bar{x} \end{aligned} \right\} \quad (23)$$

and $\bar{\mathbf{k}}$ is the curvature vector associated with bending of the midplane, given by

$$\left. \begin{aligned} (\bar{\kappa}_1) &= -\partial^2 \bar{w} / \partial \bar{x}^2 \\ (\bar{\kappa}_2) &= -\partial^2 \bar{w} / \partial \bar{y}^2 \\ (\bar{\kappa}_6) &= -2\partial^2 \bar{w} / \partial \bar{x} \partial \bar{y} \end{aligned} \right\} \quad (24)$$

Since the midplane strains and curvatures belong to the laminate, they do not change from one lamina to another, and therefore the bar over these quantities is not required and henceforth will be dispensed with.

7.0 Force-Displacement Relations for a Laminate

Equation (22) shows that the strain at a point is the sum of two components, (i) midplane extensional strain, and (ii) midplane curvature. Figure 4. depicts the displacement, strain, modulus, and stress field with respect to the \bar{x}, \bar{z} plane at an arbitrary \bar{y} location. The displacement and

strain fields are shown together in Fig.4b, while in fact, one is a scalar times the other. In accordance with the Kirchhoff-Love assumption, the displacement field is linear across the laminate thickness. As a result of the jump discontinuity of modulus from lamina to lamina (Fig.4c), the stress field also has a jump discontinuity from lamina to lamina (Fig.4d).

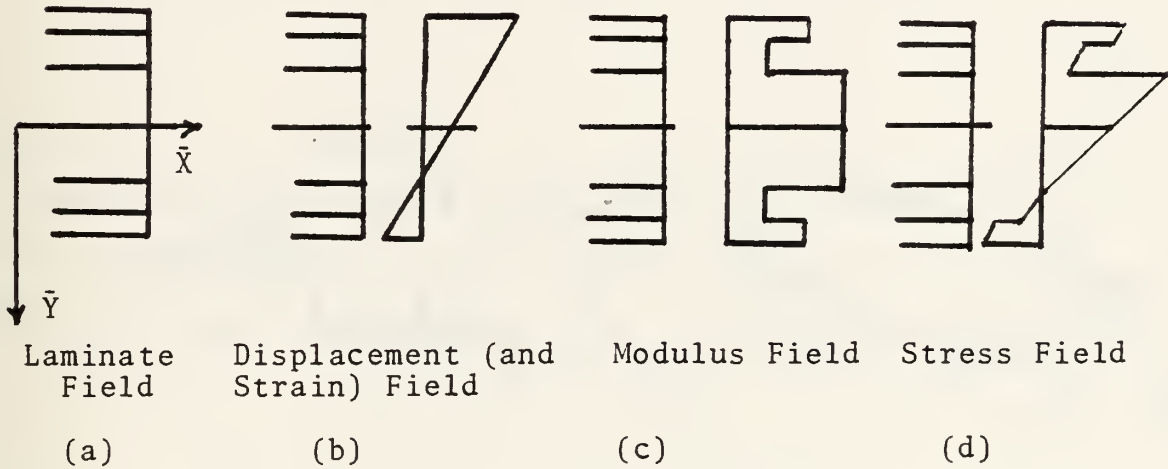


Figure 4. Displacement, Strain, Modulus and Stress Distributions

The laminate forces \bar{N} and moments \bar{M} associated with a laminate are obtained by integrations of the stress field over the laminate thickness. The extensional laminate forces are given by,

$$\begin{bmatrix} \bar{N}_1 \\ \bar{N}_2 \\ \bar{N}_6 \end{bmatrix} = \int_{-\frac{t}{2}}^{\frac{t}{2}} \begin{bmatrix} \bar{\sigma}_1 \\ \bar{\sigma}_2 \\ \bar{\sigma}_6 \end{bmatrix} d\bar{z} \quad \text{or} \quad \bar{N} = \int \bar{\sigma} d\bar{z} \quad (25)$$

and the laminate moments are given by,

$$\begin{bmatrix} \bar{M}_1 \\ \bar{M}_2 \\ \bar{M}_6 \end{bmatrix} = \int_{-\frac{t}{2}}^{\frac{t}{2}} \begin{bmatrix} \bar{\sigma}_1 \\ \bar{\sigma}_2 \\ \bar{\sigma}_6 \end{bmatrix} \bar{z} d\bar{z} \quad \bar{M} = \int \bar{\sigma} \bar{z} d\bar{z} \quad (26)$$

where t is the plate thickness. The laminate forces and moments are shown in Figure 5. Components N_1 and N_2 are the normal components of force, and N_6 is the shear component. M_1 and M_2 are bending moments, and M_6 is the twisting moment.

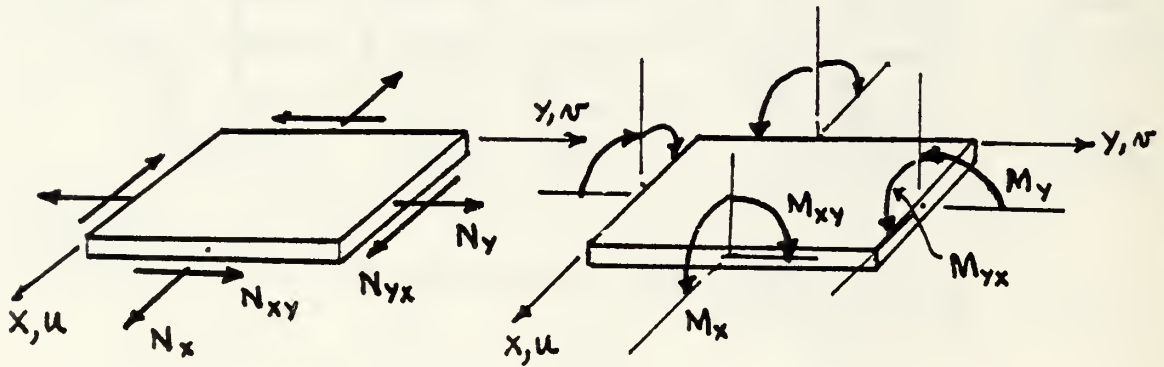


Figure 5. Laminate Forces and Moments

Since the stress field across the laminate thickness is the sum of the lamina stresses,

$$\bar{\sigma} = \sum_{i=1}^n \bar{\sigma}_i$$

equations (25) and (26) become

$$\bar{N} = \sum_{i=1}^n \int_{z_{i-1}}^{z_i} \bar{\sigma}_i d\bar{z} \quad (27)$$

and

$$\bar{M} = \sum_{i=1}^n \int_{z_{i-1}}^{z_i} \bar{\sigma}_i \bar{z} d\bar{z} \quad (28)$$

where z_{i-1} and z_i are the upper and lower limits of integration for the i -th lamina, and n is the number of lamina comprising the laminate.

Substituting the expressions for lamina stresses in terms of strains, Eq's.(16), into the previous Eq's.(27) and (28) gives the laminate forces and moments in terms of strains as,

$$\bar{N} = \sum_{i=1}^n \int_{z_{i-1}}^{z_i} \bar{Q}_i (\bar{\epsilon}_i - \bar{\alpha}_i \theta_i) d\bar{z} \quad (29)$$

and

$$\bar{M} = \sum_{i=1}^n \int_{z_{i-1}}^{z_i} \bar{Q}_i (\bar{\epsilon}_i - \bar{\alpha}_i \theta_i) \bar{z} d\bar{z} \quad (30)$$

The strain in the i -th lamina is given by Eq.(22),

$$\bar{\epsilon}_i = \bar{\epsilon}_0 + \bar{z} \bar{\kappa} \quad (31)$$

where $\bar{\epsilon}_0$ and $\bar{\kappa}$ are the midplane extensional strain and midplane curvature vectors respectively, and are constant from one lamina to another. Substituting Eq.(31) into Eq's.(29) and (30) gives,

$$\begin{aligned} \bar{N} &= \sum_{i=1}^n \int_{z_{i-1}}^{z_i} \bar{Q}_i (\bar{\epsilon}_0 + \bar{z} \bar{\kappa} - \bar{\alpha}_i \theta_i) d\bar{z} \\ \bar{M} &= \sum_{i=1}^n \int_{z_{i-1}}^{z_i} \bar{Q}_i (\bar{\epsilon}_0 + \bar{z} \bar{\kappa} - \bar{\alpha}_i \theta_i) \bar{z} d\bar{z} \end{aligned}$$

Since $\bar{\epsilon}_0$ and $\bar{\kappa}$ do not vary with \bar{z} , and $\bar{Q}_i, \bar{\alpha}_i$, and θ_i are constant for a lamina, the previous equations become,

$$\begin{aligned} \bar{N} &= \sum_{i=1}^n \bar{Q}_i [(\bar{\epsilon}_0 - \bar{\alpha}_i \theta_i) \int_{z_{i-1}}^{z_i} d\bar{z} + \bar{\kappa} \int_{z_{i-1}}^{z_i} \bar{z} d\bar{z}] \\ \bar{M} &= \sum_{i=1}^n \bar{Q}_i [(\bar{\epsilon}_0 - \bar{\alpha}_i \theta_i) \int_{z_{i-1}}^{z_i} \bar{z} d\bar{z} + \bar{\kappa} \int_{z_{i-1}}^{z_i} \bar{z}^2 d\bar{z}] \end{aligned}$$

Upon integration, we obtain,

$$\bar{N} = \sum_{i=1}^n \bar{Q}_i (\bar{\epsilon}_0 - \bar{\alpha}_i \theta_i) (\bar{z}_i - \bar{z}_{i-1}) + \sum_{i=1}^n \bar{Q}_i \bar{\kappa} \left(\frac{\bar{z}_i^2 - \bar{z}_{i-1}^2}{2} \right) \quad (32)$$

and

$$\bar{\tilde{M}} = \sum_{i=1}^n \bar{\tilde{Q}}_i (\bar{\tilde{e}}_o - \bar{\tilde{\alpha}}_i \theta_i) \frac{(\bar{z}_i^2 - \bar{z}_{i-1}^2)}{2} + \sum_{i=1}^n \bar{\tilde{Q}}_i \bar{\tilde{\kappa}}_i \frac{(\bar{z}_i^3 - \bar{z}_{i-1}^3)}{6} \quad (33)$$

Defining the matrices,

$$\bar{\tilde{A}} = \sum_{i=1}^n \bar{\tilde{Q}}_i (\bar{z}_i - \bar{z}_{i-1}) \quad (34)$$

$$\bar{\tilde{B}} = \sum_{i=1}^n \bar{\tilde{Q}}_i (\bar{z}_i^2 - \bar{z}_{i-1}^2) / 2 \quad (35)$$

$$\bar{\tilde{D}} = \sum_{i=1}^n \bar{\tilde{Q}}_i (\bar{z}_i^3 - \bar{z}_{i-1}^3) / 6 \quad (36)$$

and vectors,

$$\bar{\tilde{N}}_T = \sum_{i=1}^n \bar{\tilde{Q}}_i \bar{\tilde{\alpha}}_i \theta_i (\bar{z}_i - \bar{z}_{i-1}) \quad (37)$$

$$\bar{\tilde{M}}_T = \sum_{i=1}^n \bar{\tilde{Q}}_i \bar{\tilde{\alpha}}_i \theta_i (\bar{z}_i^2 - \bar{z}_{i-1}^2) / 2 \quad (38)$$

equations (32) and (33) become,

$$\bar{\tilde{N}} = \bar{\tilde{A}} \bar{\tilde{e}}_o + \bar{\tilde{B}} \bar{\tilde{\kappa}} - \bar{\tilde{N}}_T \quad (39)$$

$$\bar{\tilde{M}} = \bar{\tilde{B}} \bar{\tilde{e}}_o + \bar{\tilde{D}} \bar{\tilde{\kappa}} - \bar{\tilde{M}}_T \quad (40)$$

Equations (39) and (40) are the "force-displacement" relations for a laminate.

The $\bar{\tilde{N}}_T$ and $\bar{\tilde{M}}_T$ vectors are the so called thermal force vector and thermal moment vector respectively, as they result from a thermal environment. If the temperature of each lamina is equal to the stress free temperature (i.e., the curing temperature), then θ_i is zero for each lamina, and the thermal vectors are equal to the zero vector. In this case, Eq's. (39) and (40) reduce to the force-displacement relations for a laminate not subjected to thermal effects.

The $\bar{\tilde{A}}$ matrix relates inplane forces to inplane strains, and the $\bar{\tilde{D}}$ matrix relates moments to curvature. The $\bar{\tilde{B}}$ matrix

couples inplane forces to curvature, as well as moments to inplane strains. In the special, albeit common, case of symmetric laminates, the contribution to $\bar{\underline{\underline{B}}}$ from a lamina above the midplane is cancelled by the contribution from a counterpart lamina below the midplane, and as a result, the $\bar{\underline{\underline{B}}}$ matrix is the zero matrix, i.e., $\bar{\underline{\underline{B}}} = \underline{\underline{0}}$. Thus for symmetric laminates,

$$\bar{\underline{\underline{N}}}_T = \bar{\underline{\underline{A}}} \bar{\underline{\underline{e}}}_0 - \bar{\underline{\underline{N}}}_T \quad (41)$$

$$\bar{\underline{\underline{M}}}_T = \bar{\underline{\underline{D}}} \bar{\underline{\underline{\kappa}}} - \bar{\underline{\underline{M}}}_T \quad (42)$$

and therefore the extensional and bending behaviors are uncoupled. If the laminate is symmetric and the temperature distribution is symmetric with respect to the midplane, then from Eq.(38), the thermal moment vector is equal to the zero vector and Eq.(42) reduces to

$$\bar{\underline{\underline{M}}} = \bar{\underline{\underline{D}}} \bar{\underline{\underline{\kappa}}} \quad (43)$$

8.0 Stress-Force Relations for a Lamina in a Symmetric Laminate

In order to determine whether a ply has failed, it is necessary to determine the stresses in each ply. The development of the stress-force relations for a lamina, which follows, is restricted to symmetric laminates. This is not a severe restriction since symmetric laminates are in most common use. Solving Eq's.(41) and (42) for strains and curvatures gives,

$$\bar{\underline{\underline{e}}}_0 = \bar{\underline{\underline{A}}}^{-1}(\bar{\underline{\underline{N}}} + \bar{\underline{\underline{N}}}_T) \quad (44)$$

and

$$\bar{\underline{\underline{\kappa}}} = \bar{\underline{\underline{D}}}^{-1}(\bar{\underline{\underline{M}}} + \bar{\underline{\underline{M}}}_T) \quad (45)$$

The lamina strains are related to the midplane strains and curvatures of the laminate by Eq.(31),

$$\bar{\underline{\underline{e}}}_i = \bar{\underline{\underline{e}}}_0 + \bar{\underline{\underline{z}}} \bar{\underline{\underline{\kappa}}} \quad (31)$$

Substitution of Eq's.(44) and (45) into Eq.(31) gives,

$$\bar{\epsilon}_i = \bar{A}^{-1}(\bar{N} + \bar{N}_T) + \bar{z}_i \bar{D}^{-1}(\bar{M} + \bar{M}_T) \quad (46)$$

where \bar{z}_i is the distance from the laminate midplane to the middle of the i-th lamina, that is,

$$\bar{z}_i = (z_i + z_{i-1})/2 \quad (47)$$

Lamina stresses with respect to laminate axes are related to lamina strains in laminate axes by Eq.(16) as,

$$\bar{\sigma}_i = \bar{Q}_i(\bar{\epsilon}_i - \bar{\alpha}_i \theta_i) \quad (16)$$

Finally then, substitution of Eq.(46) into Eq.(16) gives the lamina stresses, with respect to laminate axes, in terms of laminate inplane forces \bar{N} , laminate moments \bar{M} , thermal forces \bar{N}_T , and thermal moments \bar{M}_T as,

$$\bar{\sigma}_i = \bar{Q}_i[\bar{A}^{-1}(\bar{N} + \bar{N}_T) + \bar{z}_i \bar{D}^{-1}(\bar{M} + \bar{M}_T) - \bar{\alpha}_i \theta_i] \quad (48)$$

Equation (48) takes on special forms for particular cases; a few of which are presented below.

- a) Temperature free effects occur when the plate is at a uniform temperature equal to the curing temperature of laminate. In this case, $\theta_i = 0$, and therefore \bar{N}_T and \bar{M}_T are equal to the zero vector, and Eq.(48) reduces to,

$$\bar{\sigma}_i = \bar{Q}_i(\bar{A}^{-1}\bar{N} + \bar{z}_i \bar{D}^{-1}\bar{M}) \quad (49)$$

In the absence of lateral loads on the laminate, $\bar{M} = 0$, and Eq.(49) becomes,

$$\bar{\sigma} = \bar{Q}\bar{A}^{-1}\bar{N} \quad (50)$$

- b) Absence of moments occurs when the laminate is not subjected to lateral loads. In this case, $\bar{M} = 0$, and

equation (48) reduces to,

$$\bar{\sigma}_i = \bar{Q}_i [\bar{A}^{-1}(\bar{N} + \bar{N}_T) + \bar{z}_i \bar{D}^{-1} \bar{M}_T - \bar{\alpha}_i \theta_i] \quad (51)$$

If the temperature field is symmetric with respect to the midplane of the laminate, then \bar{M}_T is the zero vector and Eq.(51) becomes,

$$\bar{\sigma}_i = \bar{Q}_i [\bar{A}^{-1}(\bar{N} + \bar{N}_T) - \bar{\alpha}_i \theta_i] \quad (52)$$

- c) Thermal effects only occur when the laminate is not subjected to inplane and lateral loads. In this case $\bar{N} = \bar{M} = 0$, and Eq.(48) reduces to

$$\bar{\sigma}_i = \bar{Q}_i [\bar{A}^{-1} \bar{N}_T + \bar{z}_i \bar{D}^{-1} \bar{M}_T - \bar{\alpha}_i \theta_i] \quad (53)$$

If the temperature field is symmetric with respect to the laminate midplane, then $\bar{M}_T = 0$, and Eq.(53) gives,

$$\bar{\sigma}_i = \bar{Q}_i [\bar{A}^{-1} \bar{N}_T - \bar{\alpha}_i \theta_i] \quad (54)$$

9.0 Failure Criterion

When a composite plate is subjected to a load and/or thermal environment, stresses develop in each lamina in accordance with Eq.(48). Depending upon the magnitude of the lamina stresses, one or more lamina may fail. When a lamina fails, there are two effects. First, if we assume brittle failure, then a lamina will relinquish its load carrying capacity upon failure, and there will be a redistribution of load to the remaining non failed lamina. Secondly, when a lamina fails, the stiffness matrices \bar{A} and \bar{D} change in accordance with Eq's.(34) and (36), and thus the redistribution of stresses in the non failed lamina is not simply a change in magnitude, but rather a change in the stress state.

The hypothesis of a brittle failure of a lamina is an

idealization of the actual behavior. Actually, lamina have some ductility and therefore continue to carry some fraction of load, and contribute some stiffness to the laminate upon failure. Here we assume a failed lamina serves no useful purpose after it's failure.

As will be demonstrated by illustrative examples in section 11, failure of a ply does not necessarily mean that the laminate has failed. There we will show that a cross ply laminate subjected to uniaxial loading does not fail upon failure of the first ply, and therefore the magnitude of the load can be increased beyond first ply failure. We also show that under uniaxial loading, angle ply laminates fail when the first ply fails.

There is no way to establish the validity of any failure criterion on theoretical grounds. All that can be done is to show that a failure criterion gives analytical results which agree, for a wide class of problems, with experimental results. There are several failure theories which give good results and are in common use. The most general of the macroscopic failure criteria, which includes many of the other criteria as special cases, is the Tsai-Wu criterion, Ref.(2). It is the failure criterion adopted in this work.

According to the Tsai-Wu criterion, a lamina fails when the general quadratic equation (in tensor form),

$$F_{ij}\sigma_i\sigma_j + F_i\sigma_i = 1 \quad i,j=1,2,,,6 \quad (55)$$

is satisfied. In Eq.(55), F_i and F_{ij} are strength tensors of second and fourth order, and the equation is in contracted notation. In the case of plane stress, the stress state for composite plates, Eq.(55) in terms of natural coordinates of a lamina, becomes

$$F_{11}\sigma_1^2 + 2F_{12}\sigma_1\sigma_2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + F_1\sigma_1 + F_2\sigma_2 = 1 \quad (56)$$

Equation (56) is applied to each lamina. If the left hand

side of Eq.(56) is less than unity, then the lamina has not failed. If the left hand side is greater than unity, then the lamina has failed. Failure of a lamina is imminent (impending) when the left hand side is equal to unity, i.e., when Eq.(56) is satisfied identically.

In order to use Eq.(56) directly, that is without modification, it is necessary to use lamina stresses with respect to lamina axes. The use of lamina stresses with respect to laminate axes would require the transformation of the second and fourth order tensors, F_i and F_{ij} , to laminate axes. Although this could be done, it is more expeditious to transform the lamina stresses with respect to laminate axes, given by Eq's.(48), into lamina stresses with respect to lamina axes. The transformation is accomplished by the use of Eq.(14). Thus, we obtain,

$$\begin{aligned}(\sigma_1)_i &= (\bar{\sigma}_1)_i c_i^2 + (\bar{\sigma}_2)_i s_i^2 + 2(\bar{\sigma}_6)_i c_i s_i \\(\sigma_2)_i &= (\bar{\sigma}_1)_i s_i^2 + (\bar{\sigma}_2)_i c_i^2 - 2(\bar{\sigma}_6)_i c_i s_i \\(\sigma_6)_i &= -(\bar{\sigma}_1)_i c_i s_i + (\bar{\sigma}_2)_i c_i s_i + (\bar{\sigma}_6)_i (c_i^2 - s_i^2)\end{aligned}\tag{57}$$

where $c_i = \cos\phi_i$, $s_i = \sin\phi_i$, and i denotes the lamina.

Five of the six strength parameters of Eq.(56) may be determined by a series of simple experiments as follows. A tension test of a lamina along it's fiber direction gives the longitudinal tension strength, X . A compression test along the fiber axis gives the longitudinal compressive strength, X' . Tension and compressive tests in the transverse direction (i.e., orthogonal to the fiber), gives the transverse tension and compressive strengths, Y and Y' , respectively. A shear test of the lamina gives it's shear strength, S . The strength parameters in Eq.(56) are related to the experimentally determined strengths as follows. Applying Eq.(56) to each of the five stress states associated with the five tests gives five simultaneous equations which may

be solved for $F_{11}, F_{22}, F_{66}, F_1$, and F_2 . For example, Eq.(56) for the longitudinal tension test becomes,

$$F_{11}X^2 + F_1X = 1$$

and for the longitudinal compression test, Eq.(56) is,

$$F_{11}X'^2 - F_1X' = 1$$

Solving these equations for F_{11} and F_1 gives the first two of Eq's.(58). The remaining strength parameters are obtained in a similar way.

$$\begin{aligned} F_{11} &= (1/XX') \\ F_1 &= (1/X) - (1/X') \\ F_{22} &= (1/YY') \\ F_2 &= (1/Y) - (1/Y') \\ F_{66} &= (1/S^2) \end{aligned} \tag{58}$$

The remaining strength parameter F_{12} is obtained on mathematical grounds. In order for the failure to be a closed curve, the quadratic equation must be an ellipse. This leads to the condition (see Ref.(3) for details),

$$-1 < F_{12}^2/F_{11}F_{22} < 1$$

If the Tsai-Wu criterion is required to be a generalization of the von Mises criterion, then $(F_{12}^2/F_{11}F_{22})$ must be equal to -0.5. This condition leads to,

$$F_{12} = -0.5(F_{11}F_{22})^{1/2} \tag{59}$$

Some typical values of lamina strengths are presented in Appendix A.

10.0 Strength Ratio and Failure Analysis

As discussed in the previous section, the Tsai-Wu criterion of Eq.(56) can be used to determine whether or not a

ply has failed by evaluation of the left hand side of the equation. By introducing a parameter, called the strength ratio, the criterion can be used to determine when a ply fails, the sequence of ply failure, and the load for laminate failure.

Given the temperature field of a laminate, T_i , and the relative magnitudes of the components of the laminate "load" vectors, \bar{N} and \bar{M} , we seek to determine the allowable load vectors, \bar{N}_a and \bar{M}_a , which will bring a lamina to imminent failure. Since the laminate stiffness matrices, \bar{A} and \bar{D} , change with every ply failure, a cyclic (iterative) procedure is required to trace the sequence of ply failure.

The m-th cycle determines the m-th ply which will fail. During the m-th cycle, the strength ratio of the i-th ply is defined as,

$$R_{i(m)} = \bar{N}_{1a} / \bar{N}_1 \quad (60)$$

This ratio is the same for all components of \bar{N} and \bar{M} , that is, $\bar{N}_{ja} / \bar{N}_j = \bar{M}_{ja} / \bar{M}_j$, for $j=1,2,6$. The allowable stresses in a lamina during the m-th cycle, with respect to laminate axes, are given by Eq's.(48),

$$(\bar{\sigma}_{ia})_{(m)} = \bar{Q}_i [\bar{A}_{(m)}^{-1} (\bar{N}_{a(m)} + \bar{N}_T) + \bar{z}_i \bar{D}_{(m)}^{-1} (\bar{M}_{a(m)} + \bar{M}_T) - \bar{\alpha}_i \theta_i] \quad (48)$$

where the 'a' subscript denotes allowable, the i subscript denotes the i-th lamina, the m subscript denotes the m-th cycle, and $\bar{A}_{(m)}$ and $\bar{D}_{(m)}$ are the stiffness matrices of the m-th cycle laminate. In Eq.(48), i takes on the values of only the non failed lamina. The m-th cycle laminate is comprised of the lamina which have not failed during the previous (m-1) cycles.

It is convenient to partition the lamina stresses into two parts; one part due to the \bar{N} and \bar{M} loads, and another part due to the \bar{N}_T and \bar{M}_T thermal vectors. Denoting the

stresses due to loads by $\bar{\sigma}_{iL}$, and the stresses due to temperature by $\bar{\sigma}_{iT}$, we have,

$$\bar{\sigma}_{ia(m)} = \bar{\sigma}_{iL(m)} + \bar{\sigma}_{iT(m)} \quad (61)$$

where the load stresses $\bar{\sigma}_{iL(m)}$, and thermal stresses $\bar{\sigma}_{iT(m)}$ are given by,

$$\bar{\sigma}_{iL(m)} = \bar{Q}_i [\bar{A}_{(m)}^{-1} \cdot \bar{N}_{a(m)} + \bar{z}_i \bar{D}_{(m)}^{-1} \cdot \bar{M}_{a(m)}] \quad (62)$$

$$\bar{\sigma}_{iT(m)} = \bar{Q}_i [\bar{A}_{(m)}^{-1} \cdot \bar{N}_T + \bar{z}_i \bar{D}_{(m)}^{-1} \cdot \bar{M}_T] \quad (63)$$

Using Eq's.(57), the above lamina stresses with respect to laminate axes are transformed into lamina stresses with respect to lamina axes for use in Eq.(56). Substituting lamina stresses with respect to lamina axes into the Tsai-Wu criterion of Eq.(56) gives a quadratic equation for the i -th lamina and m -th cycle,

$$a_{i(m)} R_{i(m)}^2 + b_{i(m)} R_{i(m)} + c_{i(m)} = 0 \quad (64)$$

where the coefficients are,

$$a_{i(m)} = [F_{11} \sigma_{1L}^2 + 2F_{12} \sigma_{1L} \sigma_{2L} + F_{22} \sigma_{2L}^2 + F_{66} \sigma_{6L}^2]_{i(m)} \quad (65)$$

$$b_{i(m)} = [2(F_{11} \sigma_{1L} \sigma_{1T} + F_{12} \sigma_{1L} \sigma_{2T} + F_{12} \sigma_{2L} \sigma_{1T} + F_{22} \sigma_{2L} \sigma_{2T} + F_{66} \sigma_{6L} \sigma_{6T}) + F_{11} \sigma_{1L} + F_{22} \sigma_{2L}]_{i(m)} \quad (66)$$

$$c_{i(m)} = [F_{11} \sigma_{1T}^2 + 2F_{12} \sigma_{1T} \sigma_{2T} + F_{22} \sigma_{2T}^2 + F_{66} \sigma_{6T}^2 + F_{11} \sigma_{1T} + F_{22} \sigma_{2T}]_{i(m)} - 1 \quad (67)$$

The strength ratios $R_{i(m)}$ for each of the lamina comprising the m-th cycle laminate are obtained as the roots of Eq. (64). The lamina with the smallest absolute strength ratio is the lamina which will fail during the m-th cycle. Thus the load factor to bring the m-th ply to failure is,

$$R_{(m)}^* = \text{Min}[\text{Abs}(R_{i(m)})] \quad (68)$$

Laminate failure occurs when all lamina have failed. The strength ratio (or load factor) for the laminate to fail, denoted by R_u , is the maximum of the $R_{(m)}^*$ load factors, that is,

$$R_u = \text{Max}[R_{(m)}^*] \quad (69)$$

If the maximum strength ratio is obtained for $m < n$, where n is the number of plies comprising the laminate, then the laminate fails after m plies have failed. The allowable loads that the laminate can sustain without failure, for a given temperature distribution, are obtained from Eq.(60) as,

$$\begin{aligned} \tilde{N}_a &= R_u \tilde{N} \\ \tilde{M}_a &= R_u \tilde{M} \end{aligned} \quad (70)$$

Illustrative examples are presented in section 11.

11.0 Results and Conclusions

An analysis for the prediction of the strength of fiber reinforced composite plates subjected to loads and thermal environment, described in this report, has resulted in a computer program for implementation of the analysis. A listing of the program, as well as directions for it's use, is given in appendices B and C.

Several analyses have been carried out in order to compare the results of the present analysis to other analytical results (ref.(4)), as well as experimental results. The failure criterion used in Ref.(4) is the Tsai-Hill criterion.

Figures 6 through 8 present some preliminary results. Figure 6 presents four sets of results for cross-ply laminates subjected to uniaxial loading:

- 1) experimental data for E-glass/epoxy from Ref.(4),
- 2) analytical results for E-glass/epoxy from Ref.(4),
- 3) analytical results for E-glass/epoxy from the present analysis, and
- 4) analytical results for T300/5208 graphite/epoxy from the present analysis.

The abscissa M , called the cross-ply ratio and defined by

$$M = \frac{\sum t_k}{\sum t_j} \quad \text{for even } j, \text{ and odd } k \quad (71)$$

is a measure of the relative quantity of 90 and 0 degree plies. It is seen that for cross ply laminates under uniaxial loading, there is excellent agreement between experimental data and analytical results for the E-glass/epoxy laminates. All results are for a curing temperature of 270 degrees Fahrenheit, and a loading environment at 70 degrees. In each case, the laminate did not fail at first ply failure. The first ply failure load is given in parentheses below the laminate failure load.

Figure 7 presents the same set of results for angle ply laminates as Figure 6 does for cross-ply laminates. In this case, the laminate has the layup $[-\phi/+\phi]$ with equal thickness plies. It can be seen that results from the present analysis for E glass/epoxy laminates are in good agreement with the experimental and analytical results of Ref. (4) for ϕ less than 30° , and ϕ greater than 60° . Tsai, Ref. (5), has a forthcoming report which shows that the Tsai-Wu criterion is in good agreement with experimental results for T300/5208 graphite/epoxy angle ply laminates.

Figure 8 shows the effect of temperature on laminate failure for $\pm 45^\circ$ angle ply laminates under uniaxial loading for both E-glass/epoxy, and T300/5208 graphite epoxy laminates.

In order to gain an understanding of the effect of

temperature on laminate strength, a series of computer analyses will be undertaken. The results of this computational effort will be presented in another report.

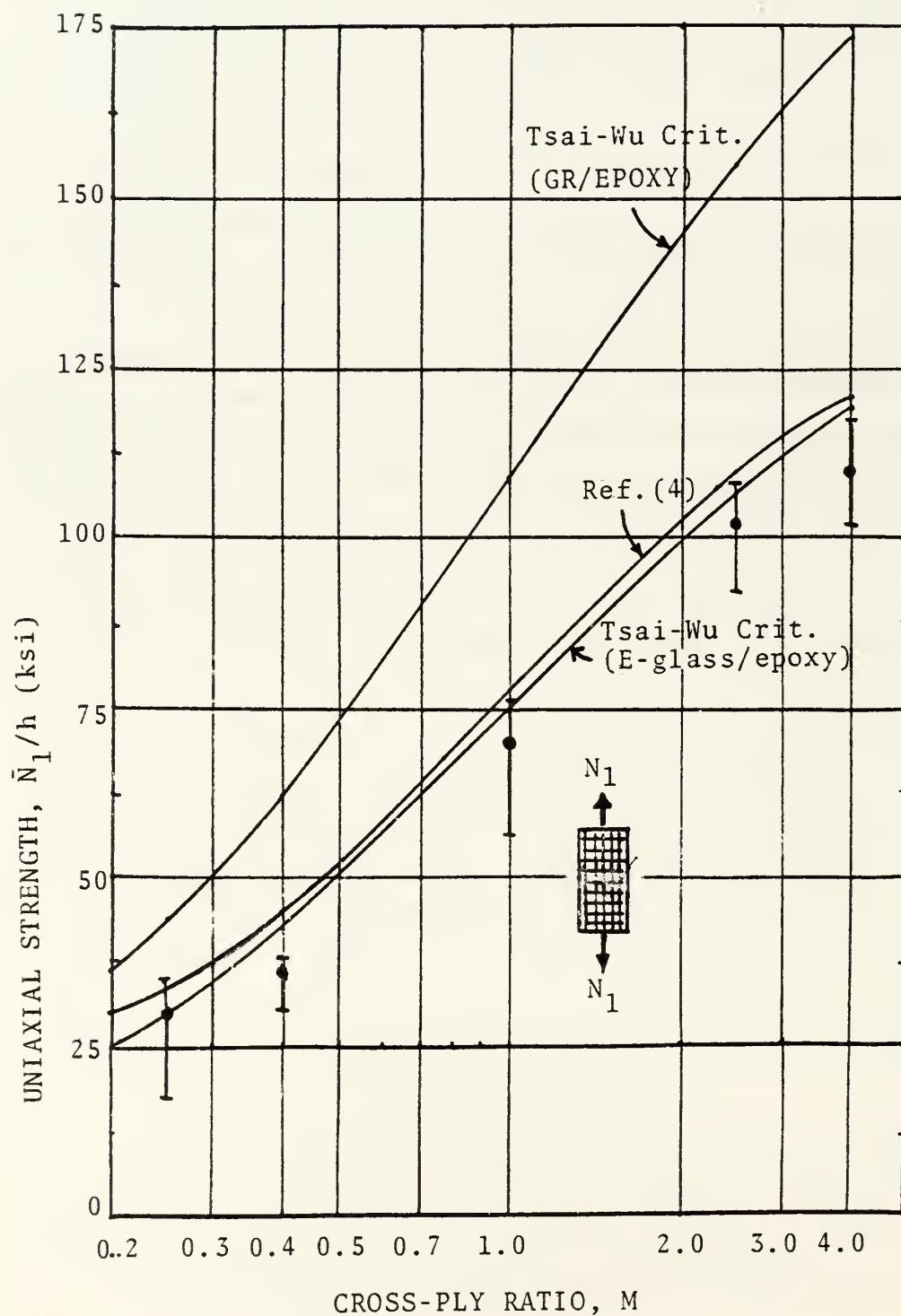


Figure 6. Experimental and Analytical Results for Cross-Ply Laminates

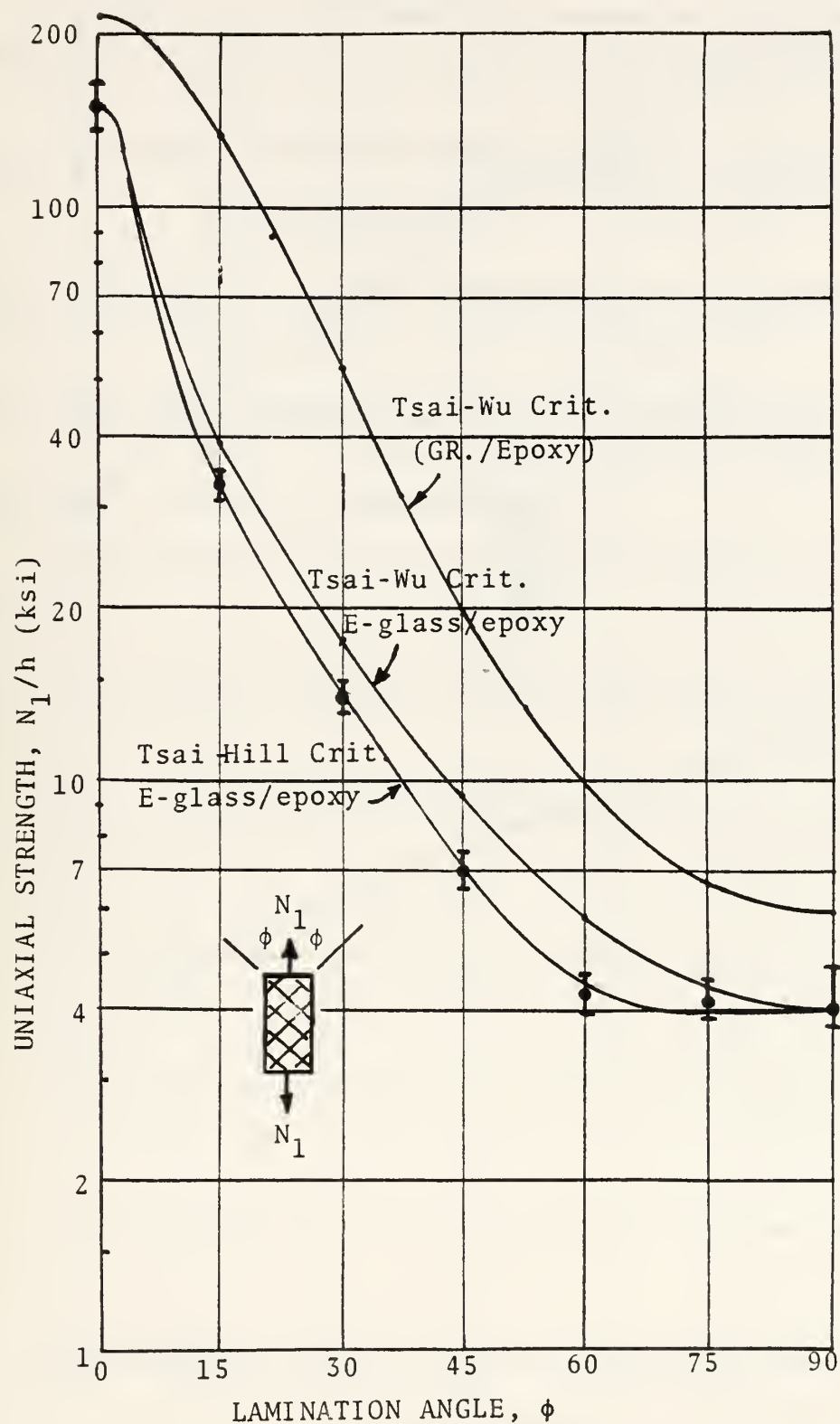


Figure 7. Experimental and Analytical Results
for Angle-Ply Laminates

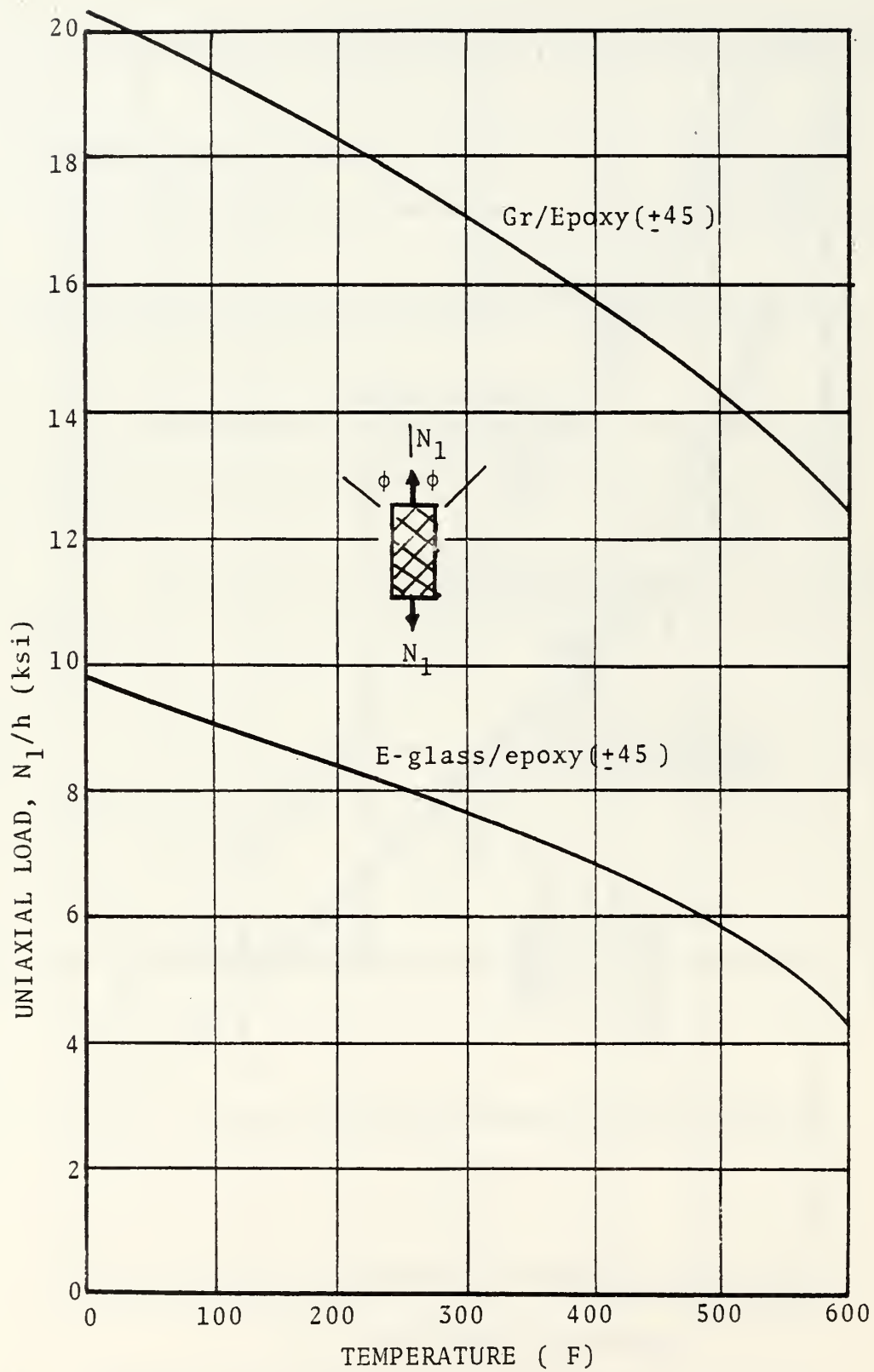


Figure.8 Load versus Temperature

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Appendix A

Typical Properties for Two Lamina

The table below presents some properties for two lamina, E-glass/epoxy, and graphite/epoxy. The information is taken from references (3) and (4).

Property	Units		E-glass/epoxy		graphite/epoxy	
	SI	(Engr'g.)				
E_x	GPa	(10^6 psi)	53.8	(7.8)	181.	(26.2)
E_y	GPa	(10^6 psi)	17.9	(2.6)	10.3	(1.49)
ν_{xy}	dimensionless		0.25	(.25)	0.28	(.28)
G_{xy}	GPa	(10^6 psi)	8.6	(1.25)	7.17	(1.04)
* ν_{yx}	dimensionless		.083	(.083)	0.16	(0.16)
α_x	1/C	(1/F)	6.3 μ	(3.5) μ	0.02 μ	(.011) μ
α_y	1/C	(1/F)	20.5	(11.4) μ	22.5 μ	(12.5) μ
X	MPa	(ksi)	1034.	(150)	1500.	(217.5)
X'	MPa	(ksi)	1034.	(150)	1500.	(217.5)
Y	MPa	(ksi)	27.6	(4.)	40.	(5.8)
Y'	MPa	(ksi)	138.	(20.)	246.	(35.7)
S	MPa	(ksi)	41.4	(6.)	68.	(9.9)
† ν_f	dimensionless				0.7	(0.7)

* calculated by EQ.(12)

† volume fraction of fiber

μ denotes micro (i.e., 10^{-6})

Appendix B

COMPUTER LISTING


```

*****
* ANALYSIS OF THE STRENGTH OF A SYMMETRIC COMPOSITE *
* PLATE DUE TO A THERMAL ENVIRONMENT *
*****

```

THIS IS THE MAIN PROGRAM WHICH READS IN THE PROPERTIES OF THE COMPOSITE, AS WELL AS THE INPLANE LOADS AND THERMAL DISTRIBUTION THROUGH THE LAMINATE. IN ADDITION, THIS PROGRAM CALLS THE SUBROUTINE CYCLE, WHICH IN TURN CALLS SUBROUTINE FAILUR.

```

COMMON ANGLE(100),Z(101),A(3,3),D(3,3),AT(3,3),DT(3,3),AINV(3,3),WSTROO
1K(50),DINV(3,3),PLYTHK(100),QBAR11(100),QBAR12(100),QBAR22(100),QBSTR00
2AR16(100),QBAR26(100),QBAR66(100),ALPHA1(100),ALPHA2(100),ALPHA3(1STR00
300),DELTEM(100),STRES1(100),STRES2(100),STRES3(100),TEM(100),STRESSSTR00
4X(100),STRESY(100),TAUXY(100),R1(100),R2(100),ZN1,ZN2,ZN3,FX,FY,FSTR00
5YY,FY,FSS,FX,FX,QQX,QQY,QQY,QSS,ALFAX,ALFAY,STFRM,STRSL1(100),STR00
6STRSL2(100),STRSL3(100),STRST1(100),STRST2(100),STRST3(100),STR00
7STRSTX(100),STRSTY(100),TAUTXY(100),STRSLX(100),STRSLY(100),TAULXYSTR00
8(100),R(70),RMAX,ZM1,ZM2,ZM3,STR00
9NPLY,ICYCLE,IFAIL,NPL,IT(100)STR00

```

1 AND 2 AFTER A NAME DENOTE THE QUANTITY IS W.R.T. THE PLATE AXES (I.E., THE GLOBAL AXES OF THE SYSTEM).
X AND Y DENOTE THE NATURAL COORDINATES OF A PLY. X IS ALONG THE FIBER AND Y IS THE TRANSVERSE DIRECTION
A AND C ARE THE INPLANE AND BENDING STIFFNESS MATRICES OF THE COMPOSITE PLATE W.R.T. LAMINATE AXES. THEY CHANGE AS PLIES FAIL.
AINV AND DINV ARE THE INVERSES OF THE A AND D MATRICES, RESPECTIVELY
ALFAX, AND ALFAY ARE THE COEFFICIENTS OF THERMAL EXPANSION W.R.T. THE NATURAL COORDINATES OF A PLY
ALPHA1(I),ALPHA2(I),ALPHA3(I) ARE THE COEFFICIENTS OF THERMAL EXPANSION OF THE I-TH PLY W.R.T. THE LAMINATE AXES.
ANGLE(I) IS THE ANGLE BETWEEN THE FIBER DIRECTION (I.E., X-AXIS) OF THE I-TH PLY AND THE 1-AXIS OF THE LAMINATE COORDINATE SYSTEM.
AT AND DT ARE THE A AND D MATRICES DIVIDED BY THE PLATE THICKNESS
DELTEM(I) IS THE TEMPERATURE DIFFERENCE OF THE I-TH PLY, AND IS EQUAL TO TEM(I)-STFRM.
FX,FX,FY,FY,FSS ARE THE FAILURE PARAMETERS IN THE TSAI-WU FAILURE CRITERION AND ARE OBTAINED FROM X, XPRIME, Y, YPRIME, SS, AND FXSTR.
IT(I) EQUALS 0 FOR NONFAILED PLIES, AND EQUALS 1 FOR FAILED PLIES
NPLY IS THE NUMBER OF PLIES ABOVE THE MIDPLANE OF A SYMMETRIC LAMINATE. THE TOTAL NUMBER OF PLIES IS 2*NPLY. THE NUMBER OF PLIES DECREASES DURING THE ANALYSIS AS PLIES FAIL.
PLYTHK(I) IS THE PLY THICKNESS OF THE I-TH PLY.
QQX,CXY,QQY,AND QSS ARE THE STIFFNESS COEFFICIENTS OF A PLY W.R.T. THE LAMINA X AND Y AXES.
QBAR11(I),QBAR12(I),QBAR22(I),QBAR16(I),QBAR26(I), AND QBAR66(I) ARE THE STIFFNESS COEFFICIENTS OF THE I-TH PLY W.R.T. THE PLATE (I.E., LAMINATE) COORDINATES.
R IS THE LOAD FACTOR (I.E., THE STRENGTH RATIO) FOR THE GIVEN TEMPERATURE DISTRIBUTION AND UNSCALED INPUT LOADING. R GREATER THAN 1. IS ASSOCIATED W. SAFE LOADING, AND R LESS THAN 1. MEANS

```

FAILURE HAS OCCURRED. FAILURE OCCURS WHEN ABS(MIN(R(I))) IS STR00560
EQUAL TO UNITY. STR00570
STRES1(I),STRES2(I),STRES3(I) ARE THE STRESSES IN THE I-TH PLY W.R.T. STR00580
THE LAMINATE AXES. STR00590
STRESX(I),STRESY(I), AND TAUXY(I) ARE THE STRESSES IN THE I-TH PLY STR00600
W.R.T. THE NATURAL (PLY) AXES, AFTER SCALING WITH THE LOAD FACTOR. STR00610
STRSL FOLLOWED BY A LAMINA OR LAMINATE "SUBSCRIPT" (I.E. STRSLX OR STR00620
STRSL1) IS THE PLY STRESS DUE TO LOAD W.R.T. LAMINA OR LAMINATE STR00630
COORDINATES BEFORE SCALING WITH THE LOAD FACTOR. STR00640
STRST FOLLOWED BY A LAMINA OR LAMINATE "SUBSCRIPT" IS THE PLY STRESS STR00650
DUE TO TEMPERATURE W.R.T. LAMINA OR LAMINATE COORDINATES STR00660
STFRM IS THE STRESS FREE TEMPERATURE STR00670
TEM(I) IS THE TEMPERATURE OF THE I-TH PLY. STR00680
THICK IS THE PLATE THICKNESS STR00690
X,XPRIME,Y,YPRIME,SS, AND FXYSTR ARE THE FAILURE PARAMETERS DETER- STR00700
MINED EXPERIMENTALLY. STR00710
Z(I) AND Z(I+1) DENOTE THE LOCATIONS OF THE BOTTOM AND TOP STR00720
SURFACES OF THE I-TH PLY, RESPECTIVELY. STR00730
ZN1,ZN2, AND ZN3 ARE THE UNSCALED INPLANE LOADS STR00740
ZNUXY,ZNUYX,EX,EY, AND G ARE THE STIFFNESS COEFFICIENTS OF A PLY STR00750
W.R.T. PLY AXES. STR00760
STR00770
READ(5,40) NPLY,ITYPE,ZNUXY,EX,EY,G,ALFAX,ALFAY,STFRM STR00780
ZNUYX=ZNUXY*EY/EX STR00790
WRITE(6,50) NPLY,ITYPE STR00800
WRITE(6,60) STR00810
WRITE(6,70) STR00820
WRITE(6,71) STR00830
WRITE(6,80) (ZNUXY,ZNUYX,EX,EY,G) STR00840
WRITE(6,90) ALFAX,ALFAY STR00850
WRITE(6,100) STFRM STR00860
WRITE(6,105) STR00870
NN=NPLY+1 STR00880
NPL=NPLY STR00890
STR00900
IF ITYPE=0 ALL PLIES ARE THE SAME THICKNESS STR00910
IF ITYPE=1 PLIES HAVE DIFFERENT THICKNESSES STR00920
STR00930
ICYCLE=1 STR00940
Z(1)=0. STR00950
IF (ITYPE.NE.0) GO TO 20 STR00960
READ(5,110) THICK STR00970
PLT=THICK/NPLY STR00980
DO 10 I=1,NPLY STR00990
IT(I)=0 STR01000
PLYTHK(I)=PLT STR01010
Z(I+1)=Z(I)+PLT STR01020
CONTINUE STR01030
CONTINUE STR01040
IF (ITYPE.NE.1) GO TO 30 STR01050
READ(5,120) (J,PLYTHK(I),I=1,NPLY) STR01060
DO 25 I=1,NPLY STR01070
Z(I+1)=Z(I)+PLYTHK(I) STR01080
IT(I)=0 STR01090
CONTINUE STR01100

```

```

30  READ (5,130) (J,ANGLE(I),TEM(I),I=1,NPLY)
    WRITE (6,65)
    WRITE (6,75) (J,ANGLE(J),PLYTHK(J),TEM(J),J=1,NPL)
    WRITE (6,85)
    WRITE (6,95) (Z(I),I=1,NN)
C
C  INPUT THE APPLIED INPLANE LOADS
C
    READ (5,140) ZN1,ZN2,ZN3
    READ (5,140) ZM1,ZM2,ZM3
    WRITE (6,210)
    WRITE (6,220) ZN1,ZN2,ZN3
    WRITE (6,230)
    WRITE (6,240) ZM1,ZM2,ZM3
    WRITE (6,245)
C
C  LAMINA STIFFNESS COEFFICIENTS IN NATURAL (X,Y) COORDINATES
C
    DENCN=1.0-ZNUXY*ZNUYX
    QXX=EX/DENOM
    QXY=(ZNUXY*EY)/DENOM
    QYY=EY/DENOM
    QSS=G
C
    WRITE (6,150)
    WRITE (6,160) QXX,QYY,QXY,QSS
C
C  CALCULATE THE COEFFICIENTS OF THE FAILURE CRITERION
C
    READ (5,170) X,XPRIME,Y,YPRIME,SS,FXWSTR
    WRITE (6,175)
    WRITE (6,180) X,XPRIME,Y,YPRIME,SS,FXWSTR
C
    FXX=1./(X*XPRIME)
    FX=(1./X)-(1./XPRIME)
    FYY=1./(Y*YPRIME)
    FY=(1./Y)-(1./YPRIME)
    FXY=FXWSTR*SQRT(FXX*FYY)
    FSS=1./(SS**2)
C
    WRITE (6,195)
    WRITE (6,190)
    WRITE (6,200) FXX,FX,FYY,FY,FSS,FXY
C
    CALL CYCLE
C
    RETURN
C
40  FORMAT (2I3,7G10.4)
50  FORMAT (1H0,2X,28HNO. OF PLYS ABOVE MIDPLANE=,I3,10X,6H1TYPE=,I3)
60  FORMAT (1H0,2X,35H1TYPE=0 FOR UNIFORM PLY THICKNESSES)
70  FORMAT (3X,35H1TYPE=1 FOR NON UNIFORM PLY THICKNESSES)
71  FCRMAT(//1H0,2X,'LAMINA ELASTIC PROPERTIES W.R.T. LAMINA COORDINAT

```



```

&ES: ')
FCRMT (1H0,2X,5HNUXY=,G9.3,2X,5HNUYX=,G9.3,2X,3HEX=,G9.4,3X,3HEY=
1,G9.4,3X,2HG=,G9.4)
FCRMT (1H0,2X,7HALPHAX=,G10.4,10X,7HALPHAY=,G10.4)
FCRMT (//1H0,2X,'THE STRESS FREE TEMPERATURE IS',G10.4)
FCRMT (//1H0,2X,'LAMINATE CONSTRUCTION:')
FCRMT (G10.4)
FCRMT (13,G10.4)
FCRMT (13,2G10.4)
FCRMT (3G10.4)
FCRMT (//1H0,2X,46HSTIFFNESS COEFFICIENTS IN NATURAL COORDINATES:
1)
FCRMT (1H0,5X,4HQXX=,G10.3,2X,4HQYY=,G10.3,2X,4HQXY=,G10.3,2X,4HQ
1SS=,G10.3)
FCRMT (6G10.4)
FCRMT (//1H0,2X,'STRENGTHS W.R.T. LAMINA AXES:')
FCRMT (1H0,4X,2HX=,G10.4,3X,7HXPRIME=,G10.4,3X,2HY=,G10.4,3X,7HYP
1RIME=,G10.4,3X,3HSS=,G10.4,3X,7HFXSTR=,G10.4)
FCRMT (1H0,7X,3HFX,11X,2HFX,11X,3HFF,10X,2HFF,10X,3HFF,10X,3HFF
1XY)
FCRMT (//1H0,2X,'TSAI-WU STRENGTH PARAMETERS W.R.T. LAMINA AXES:')
FCRMT (4X,G10.4,3X,G10.4,3X,G10.4,3X,G10.4,3X,G10.4,3X,G10.4)
FCRMT (//1H0,2X,'INPLANE LOADS W.R.T. LAMINATE AXES:')
FCRMT (1H0,5X,'ZN1=',G10.4,5X,'ZN2=',G10.4,5X,'ZN3=',G10.4)
FCRMT (//1H0,2X,'MOMENTS W.R.T. LAMINATE AXES:')
FCRMT (1H0,5X,'ZM1=',G10.4,5X,'ZM2=',G10.4,5X,'ZM3=',G10.4)
FCRMT (1H0,2X,'*** MULTIPLICATION OF THE ABOVE LOADS AND MOMENTS B
1Y THE STRENGTH RATIO AT LAMINATE FAILURE GIVES THE FAILURE LOADS
2AND CMMENTS')
FCRMT (1H0,9X,3HPLY,10X,5HANGLE,11X,13HPLY THICKNESS,7X,3HTEM)
FCRMT (9X,13,10X,G10.4,9X,G10.4,6X,G10.4)
FCRMT (1H0,2X,10HZ-LOCATION)
FCRMT (5X,G10.4)
END

```

SUBROUTINE CYCLE

SUBROUTINE CYCLE CONSTRUCTS THE A AND D MATRICES, AND THEN SOLVES FOR THE STRESSES IN EACH PLY. AFTER THE STRESSES ARE CALCULATED, SUBROUTINE FAILUR IS CALLED TO DETERMINE WHICH, IF ANY, PLIES HAVE FAILED. SUBROUTINE FAILUR IN TURN CALLS SUBROUTINE CYCLE UNTIL ALL PLIES WHICH WILL FAIL DUE TO THE LOAD AND THERMAL ENVIRONMENT HAVE FAILED.

```

COMMON ANGLE(100),Z(101),A(3,3),D(3,3),AT(3,3),DT(3,3),AINV(3,3),W
1K(50),DINV(3,3),PLYTHK(100),QBAR11(100),QBAR12(100),QBAR22(100),QB
2AR16(100),QBAR26(100),QBAR66(100),ALPHA1(100),ALPHA2(100),ALPHA3(1
300),CELTEM(100),STRES1(100),STRES2(100),STRES3(100),TEM(100),STRES
4X(100),STRESY(100),TAUXY(100),R1(100),R2(100),ZN1,ZN2,ZN3,FX,FX,F
5YY,FY,FSS,FX,Y,QXX,QXY,QYY,QSS,ALFAX,ALFAY,STFRM,STRSL1(100),
6STRSL2(100),STRSL3(100),STRST1(100),STRST2(100),STRST3(100),
7STRSTX(100),STRSTY(100),TAUTXY(100),STRSLX(100),STRSLY(100),TAULXY
8(100),R(70),RMAX,ZM1,ZM2,ZM3,
9NPLY,ICYCLE,IFAIL,NPL,IT(100)

```

[illegible]


```

P=PLYTHK(IPLY)
DTM=DELTEM(IPLY)
A1=ALPHA1(IPLY)
A2=ALPHA2(IPLY)
A3=ALPHA3(IPLY)
ZNT1=ZNT1+P*DTM*(QBAR11(IPLY)*A1+QBAR12(IPLY)*A2+QBAR16(IPLY)*A3)
ZNT2=ZNT2+P*DTM*(QBAR12(IPLY)*A1+QBAR22(IPLY)*A2+QBAR26(IPLY)*A3)
ZNT3=ZNT3+P*DTM*(QBAR16(IPLY)*A1+QBAR26(IPLY)*A2+QBAR66(IPLY)*A3)

```

CONSTRUCTION OF LAMINATE A AND D MATRICES FROM LAMINA MATRICES

```

Z1=Z(IPLY+1)
Z2=Z(IPLY)
THICK=2.*(Z1-Z2)
RK=2.*((Z1**3)-(Z2**3))/3.

```

```

A(1,1)=A(1,1)+THICK*QBAR11(IPLY)
A(1,2)=A(1,2)+THICK*QBAR12(IPLY)
A(1,3)=A(1,3)+THICK*QBAR16(IPLY)
A(2,1)=A(1,2)
A(2,2)=A(2,2)+THICK*QBAR22(IPLY)
A(2,3)=A(2,3)+THICK*QBAR26(IPLY)
A(3,1)=A(1,3)
A(3,2)=A(2,3)
A(3,3)=A(3,3)+THICK*QBAR66(IPLY)

```

```

D(1,1)=D(1,1)+RK*QBAR11(IPLY)
D(1,2)=D(1,2)+RK*QBAR12(IPLY)
D(1,3)=D(1,3)+RK*QBAR16(IPLY)
D(2,1)=D(1,2)
D(2,2)=D(2,2)+RK*QBAR22(IPLY)
D(2,3)=D(2,3)+RK*QBAR26(IPLY)
D(3,1)=D(1,3)
D(3,2)=D(2,3)
D(3,3)=D(3,3)+RK*QBAR66(IPLY)

```

CONTINUE

```

ZNT1=2.*ZNT1
ZNT2=2.*ZNT2
ZNT3=2.*ZNT3

```

CALCULATE THE TOTAL INPLANE FORCES. TOTN1=ZN1+ZNT1

```

TOTN1=ZN1+ZNT1
TOTN2=ZN2+ZNT2
TOTN3=ZN3+ZNT3

```

```

WRITE(6,125)
WRITE(6,130) ZN1,ZNT1,TOTN1
WRITE(6,140) ZN2,ZNT2,TOTN2
WRITE(6,150) ZN3,ZNT3,TOTN3

```

WRITE(6,155)

```

STR02760
STR02770
STR02780
STR02790
STR02800
STR02810
STR02820
STR02830
STR02840
STR02850
STR02860
STR02870
STR02880
STR02890
STR02900
STR02910
STR02920
STR02930
STR02940
STR02950
STR02960
STR02970
STR02980
STR02990
STR03000
STR03010
STR03020
STR03030
STR03040
STR03050
STR03060
STR03070
STR03080
STR03090
STR03100
STR03110
STR03120
STR03130
STR03140
STR03150
STR03160
STR03170
STR03180
STR03190
STR03200
STR03210
STR03220
STR03230
STR03240
STR03250
STR03260
STR03270
STR03280
STR03290
STR03300

```

40

```

WRITE(6,225)
WRITE(6,230)
DO 30 I=1,NPLY
IF(IT(I).EQ.1) GO TO 30
AINT1=AINV(1,1)*ZNT1+AINV(1,2)*ZNT2+AINV(1,3)*ZNT3
AINT2=AINV(2,1)*ZNT1+AINV(2,2)*ZNT2+AINV(2,3)*ZNT3
AINT3=AINV(3,1)*ZNT1+AINV(3,2)*ZNT2+AINV(3,3)*ZNT3

```

2. ALPHA*DELTEM

```

ALTM1=ALPHA1(I)*DELTEM(I)
ALTM2=ALPHA2(I)*DELTEM(I)
ALTM3=ALPHA3(I)*DELTEM(I)

```

3. THERMAL STRESS= QBAR*(AINV*ZNT-ALPHA*DELTEM)

```

A1=AINT1-ALTM1
A2=AINT2-ALTM2
A3=AINT3-ALTM3
STRST1(I)=QBAR11(I)*A1+QBAR12(I)*A2+QBAR16(I)*A3
STRST2(I)=QBAR12(I)*A1+QBAR22(I)*A2+QBAR26(I)*A3
STRST3(I)=QBAR16(I)*A1+QBAR26(I)*A2+QBAR66(I)*A3
WRITE(6,240) I,STRST1(I),STRST2(I),STRST3(I),DELTEM(I)
CONTINUE

```

CALCULATE THE PLY STRESSES DUE TO LOADS; STRSL=Q*AINV*ZN

```

DO 35 I=1,NPLY
IF(IT(I).EQ.1) GO TO 35
AINL1=AINV(1,1)*ZN1+AINV(1,2)*ZN2+AINV(1,3)*ZN3
AINL2=AINV(2,1)*ZN1+AINV(2,2)*ZN2+AINV(2,3)*ZN3
AINL3=AINV(3,1)*ZN1+AINV(3,2)*ZN2+AINV(3,3)*ZN3
STRSL1(I)=QBAR11(I)*AINL1+QBAR12(I)*AINL2+QBAR16(I)*AINL3
STRSL2(I)=QBAR12(I)*AINL1+QBAR22(I)*AINL2+QBAR26(I)*AINL3
STRSL3(I)=QBAR16(I)*AINL1+QBAR26(I)*AINL2+QBAR66(I)*AINL3

```

CCNTINUE

CALCULATE THE STRESSES DUE TO MOMENTS

```

IF(ZM1.EQ.0..AND.ZM2.EQ.0..AND.ZM3.EQ.0.) GO TO 37
DO 36 I=1,NPLY
IF(IT(I).EQ.1) GO TO 36
ZZ=(Z(I+1)+Z(I))/2.
ZKAP1=DINV(1,1)*ZM1+DINV(1,2)*ZM2+DINV(1,3)*ZM3
ZKAP2=DINV(2,1)*ZM1+DINV(2,2)*ZM2+DINV(2,3)*ZM3
ZKAP3=DINV(3,1)*ZM1+DINV(3,2)*ZM2+DINV(3,3)*ZM3
DINM1=ZZ*ZKAP1
DINM2=ZZ*ZKAP2
DINM3=ZZ*ZKAP3

```

STR03860
 STR03870
 STR03880
 STR03890
 STR03900
 STR03910
 STR03920
 STR03930
 STR03940
 STR03950
 STR03960
 STR03970
 STR03980
 STR03990
 STR04000
 STR04010
 STR04020
 STR04030
 STR04040
 STR04050
 STR04060
 STR04070
 STR04080
 STR04090
 STR04100
 STR04110
 STR04120
 STR04130
 STR04140
 STR04150
 STR04160
 STR04170
 STR04180
 STR04190
 STR04200
 STR04210
 STR04220
 STR04230
 STR04240
 STR04250
 STR04260
 STR04270
 STR04280
 STR04290
 STR04300
 STR04310
 STR04320
 STR04330
 STR04340
 STR04350
 STR04360
 STR04370
 STR04380
 STR04390
 STR04400

```

C      ADD TC STRESSES DUE TO INPLANE LOADS
C      STRSL1(I)=STRSL1(I)+QBAR11(I)*DINM1+QBAR12(I)*DINM2+QBAR16(I)*DINM3
C      &3 STRSL2(I)=STRSL2(I)+QBAR12(I)*DINM1+QBAR22(I)*DINM2+QBAR26(I)*DINM3
C      &3 STRSL3(I)=STRSL3(I)+QBAR16(I)*DINM1+QBAR26(I)*DINM2+QBAR66(I)*DINM3
C      &3
36     CONTINUE
37     CONTINUE
      WRITE(6,270)
      DO 38 I=1,NPLY
      IF(IT(I).EQ.1) GO TO 38
      WRITE(6,280) I,STRSL1(I),STRSL2(I),STRSL3(I)
38     CONTINUE

C      FAILURE ANALYSIS
C      9 1. CCNVERT PLY STRESSES TO NATURAL (LAMINA) AXES
      DO 40 I=1,NPLY
      IF(IT(I).EQ.1) GO TO 40

      ANGLE1=ANGLE(I)/57.29578
      S=SIN(ANGLE1)
      S2=S**2
      C=COS(ANGLE1)
      C2=C**2
      SC=S*C
      STRSTX(I)=STRST1(I)*C2+STRST2(I)*S2+STRST3(I)*(2.*SC)
      STRSTY(I)=STRST1(I)*S2+STRST2(I)*C2-STRST3(I)*(2.*SC)
      TAUTXY(I)=SC*(STRST2(I)-STRST1(I))+STRST3(I)*(C2-S2)

      STRSLX(I)=STRSL1(I)*C2+STRSL2(I)*S2+STRSL3(I)*(2.*SC)
      STRSLY(I)=STRSL1(I)*S2+STRSL2(I)*C2-STRSL3(I)*(2.*SC)
      TAULXY(I)=SC*(STRSL2(I)-STRSL1(I))+STRSL3(I)*(C2-S2)

C      2. CALCULATE THE COEFFICIENTS OF THE QUADRATIC: A*(R(I)**2)
C      +B*R(I)+C=0.
      AA=FXX*(STRSLX(I)**2)+2.*FXY*(STRSLX(I)*STRSLY(I))+FYY*(STRSLY(I)**2)+FSS*(TAULXY(I)**2)
      B=2.*(FXX*STRSLX(I)*STRSTX(I)+FXY*STRSLX(I)*STRSTY(I)+FXY*STRSLY(I)*STRSTX(I)+FYY*STRSLY(I)*STRSTY(I)+FSS*TAULXY(I)*2*TAUTXY(I))+FX*STRSLX(I)+FY*STRSLY(I)
      C=FXX*(STRSTX(I)**2)+2.*FXY*STRSTX(I)*STRSTY(I)+FYY*(STRSTY(I)**2)+FSS*(TAUTXY(I)**2)+FX*STRSTX(I)+FY*STRSTY(I)-1.

C      3. SOLVE THE FAILURE QUADRATIC FOR THE STRENGTH RATIOS, R(I)
      CC=SQRT((B**2)-4.*AA*C)
      R1(I)=(-B+CC)/(2.*AA)
      R2(I)=(-B-CC)/(2.*AA)

```


CONTINUE

```
WRITE (6,250)
DO 41 I=1,NPLY
IF (IT(I).EQ.1) GO TO 41
WRITE (6,260) I,STRSTX(I),STRSTY(I),TAUTXY(I),R1(I),R2(I)
CONTINUE
```

```
WRITE(6,290)
DO 42 I=1,NPLY
IF (IT(I).EQ.1) GO TO 42
WRITE(6,300) I,STRSLX(I),STRSLY(I),TAULXY(I)
CONTINUE
```

CALL FAILUR

RETURN

```
FORMAT (1H1,1X,7H1CYCLE=,I3)
FORMAT (//1H0,2X,'PLY STIFFNESS COEFFICIENTS W.R.T. LAMINATE AXES:
1')
FORMAT (1H0,3X,3HPLY,3X,6HQBAR11,5X,6HQBAR22,5X,6HQBAR12,5X,6HQBAR
166,5X,6HQBAR16,5X,6HQBAR26)
FORMAT (2X,I3,6(1X,G10.3))
FORMAT(//1H0,2X,'APPLIED, THERMAL, AND TOTAL INPLANE FORCES:')
FORMAT (1H0,5X,4HZN1=,G10.4,10X,5HZNT1=,G10.4,10X,6HTCTN1=,G10.4)
FORMAT (6X,4HZN2=,G10.4,10X,5HZNT2=,G10.4,10X,6HTOTN2=,G10.4)
FORMAT (6X,4HZN3=,G10.4,10X,5HZNT3=,G10.4,10X,6HTOTN3=,G10.4)
FORMAT (//1H0,2X,'THERMAL COEFFICIENTS OF EXPANSION W.R.T. LAMINAT
1E AXES:')
FORMAT (1H0,3X,3HPLY,11X,6HALPHA1,14X,6HALPHA2,14X,6HALPHA6)
FORMAT (3X,I3,10X,G10.4,10X,G10.4,10X,G10.4)
FORMAT (//1H0,12X,8HA-MATRIX,35X,8HD-MATRIX)
FORMAT (1H0,1X,G11.4,1X,G11.4,1X,G11.4,8X,G11.4,1X,G11.4,1X,G11.4)
FORMAT (2X,G11.4,1X,G11.4,1X,G11.4,8X,G11.4,1X,G11.4,1X,G11.4)
FORMAT (//1H0,5X,16HPLATE THICKNESS=,G10.4)
FORMAT (//1H0,9X,18HA-MATRIX/THICKNESS,25X,18HD-MATRIX/THICKNESS)
FORMAT (1H0,9X,16HA-INVERSE MATRIX,30X,16HD-INVERSE MATRIX)
FORMAT(//1H0,2X,'STRESSES IN LAMINA AND LAMINATE COORDINATES BEFOR
&E MULTIPLICATION WITH THE LOAD FACTOR:')
FORMAT (1H0,3X,3HPLY,6X,6HSTRST1,12X,6HSTRST2,14X,6HSTRST3,9X,6HDE
1LTEM)
FORMAT (4X,I3,7X,G10.4,8X,G10.4,9X,G10.4,5X,G10.4)
FORMAT (1H0,3X,3HPLY,6X,6HSTRSTX,9X,6HSTRSTY,11X,6HTAUTXY,12X,2HR1
1,12X,2HR2)
FORMAT (4X,I3,5X,G10.4,5X,G10.4,5X,G10.4,5X,G10.4,5X,G10.4)
FORMAT(1H0,3X,3HPLY,6X,6HSTRSL1,12X,6HSTRSL2,14X,6HSTRSL3)
FORMAT(4X,I3,7X,G10.4,8X,G10.4,9X,G10.4)
FORMAT(1H0,3X,3HPLY,6X,6HSTRSLX,9X,6HSTRSLY,11X,6HTAULXY)
FORMAT(4X,I3,5X,G10.4,5X,G10.4,5X,G10.4)
END
```

SUBRCUTINE FAILUR

STR04960
STR04970
STR04980
STR04990
STR05000
STR05010
STR05020
STR05030
STR05040
STR05050
STR05060
STR05070
STR05080
STR05090
STR05100
STR05110
STR05120
STR05130
STR05140
STR05150
STR05160
STR05170
STR05180
STR05190
STR05200
STR05210
STR05220
STR05230
STR05240
STR05250
STR05260
STR05270
STR05280
STR05290
STR05300
STR05310
STR05320
STR05330
STR05340
STR05350
STR05360
STR05370
STR05380
STR05390
STR05400
STR05410
STR05420
STR05430
STR05440
STR05450
STR05460
STR05470
STR05480
STR05490
STR05500


```

C THIS SUBROUTINE DETERMINES WHICH PLIES HAVE FAILED BY USING
C THE TSAI-WU FAILURE CRITERION. THE PLY WHICH HAS FAILED IS
C ELIMINATED AND THE ANALYSIS IS RETURNED TO SUBROUTINE CYCLE,
C WHICH RECALCULATES THE STRESSES FOR THE MODIFIED LAMINATE AND
C RETURNS TO SUBROUTINE FAILURE AGAIN. THIS PROCESS IS REPEATED
C UNTIL ALL PLIES WHICH ARE TO FAIL HAVE FAILED.

```

```

C THIS SUBROUTINE CHECKS FOR PLY FAILURE.

```

```

C COMMON ANGLE(100),Z(101),A(3,3),D(3,3),AT(3,3),DT(3,3),AINV(3,3),W
1K(50),DINV(3,3),PLYTHK(100),QBAR11(100),QBAR12(100),QBAR22(100),QB
2AR16(100),QBAR26(100),QBAR66(100),ALPHA1(100),ALPHA2(100),ALPHA3(1
300),DELTEM(100),STRES1(100),STRES2(100),STRES3(100),TEM(100),STRES
4X(100),STRESY(100),TAUXY(100),R1(100),R2(100),ZN1,ZN2,ZN3,FX,FY,F
5YY,FY,FSS,FX,QQX,QQY,QQY,QSS,ALFAX,ALFAY,STFRTM,STRSL1(100),
6STRSL2(100),STRSL3(100),STRST1(100),STRST2(100),STRST3(100),
7STRSTX(100),STRSTY(100),TAUTXY(100),STRSLX(100),STRSLY(100),TAULXY
8(100),R(70),RMAX,ZM1,ZM2,ZM3,
9NPLY,ICYCLE,IFAIL,NPL,IT(100)

```

```

C R(ICYCLE)=1.E+20

```

```

C DO 10 I=1,NPLY
C IF(IT(I).EQ.1) GO TO 10
C ABSR1=ABS(R1(I))
C ABSR2=ABS(R2(I))
C ABSR=AMIN1(ABSR1,ABSR2)
C IF(ABSR.LT.R(ICYCLE)) IFAIL=I
C IF(ABSR.LT.R(ICYCLE)) R(ICYCLE)=ABSR
10 CONTINUE

```

```

C WRITE(6,110) ICYCLE,IFAIL,R(ICYCLE)

```

```

C CALCULATE THE STRESSES AT IMPENDING PLY FAILURE

```

```

C DO 20 I=1,NPLY
C IF(IT(I).EQ.1) GO TO 20
C STRESX(I)=R(ICYCLE)*STRSLX(I)+STRSTX(I)
C STRESY(I)=R(ICYCLE)*STRSLY(I)+STRSTY(I)
C TAUXY(I)=R(ICYCLE)*TAULXY(I)+TAUTXY(I)
20 CONTINUE

```

```

C WRITE(6,100)
C WRITE(6,120)
C DO 30 I=1,NPLY
C IF(IT(I).EQ.1) GO TO 30
C WRITE(6,125) (I,STRESX(I),STRESY(I),TAUXY(I))
30 CONTINUE

```

```

C CALCULATE THE STRENGTH RATIO FOR LAMINATE FAILURE

```

```

C IF(ICYCLE.EQ.1) RMAX=R(1)
C RMAX=AMAX1(R(ICYCLE),RMAX)

```

CHECK FOR LAMINATE FAILURE

NPL=NPL-1	STR06060
IF(NPL.NE.0) GO TO 50	STR06070
WRITE(6,130) RMAX	STR06080
IF(NPL.EQ.0) STOP 1	STR06090
ICYCLE = ICYCLE+1	STR06100
IT(IFAIL)=1	STR06110
CALL CYCLE	STR06120
RETURN	STR06130
0 FORMAT(//1H0,2X,' STRESSES AT IMPENDING PLY FAILURE AFTER MULTIPLIC	STR06140
0 &ATION W. THE LOAD FACTOR:')	STR06150
1) FCRMAT(//1H0,2X,' CYCLE=',I3,5X,' IFAIL=',I3,5X,' LOAD FACTOR=',G10.4	STR06160
0 1)	STR06170
5 FCRMAT(1H0,3X,' PLY',8X,' STRESX',8X,' STRESY',10X,' TAUXY')	STR06180
0 FORMAT(4X,I3,6X,G10.4,4X,G10.4,4X,G10.4)	STR06190
0 FORMAT(////////1H0,2X,' *** THE STRENGTH RATIO FOR THE LAMINATE IS'	STR06200
0 &,G10.4)	STR06210
END	STR06220
	STR06230
	STR06240
	STR06250
	STR06260
	STR06270
	STR06280

Appendix C

Instructions for Use of the Computer Program

The following describes the sequence of input data for the computer program:

1. Read (5,40) NPLY, ITYPE, ZNUXY, EX, EY, ALFAX, ALFAY, STFRTM

40 Format (2I3, 7G10.4)

NPLY = Number of plies above the laminate midplane

ITYPE is 0 if all plies are of equal thickness;

ITYPE is 1 if plies have different thicknesses

ZNUXY (ν_{xy}) = lamina longitudinal Poissons ratio

EX (E_x) = lamina longitudinal stiffness

EY (E_y) = lamina transverse stiffness

ALFAX (α_x) = lamina longitudinal coefficient of thermal expansion

ALFAY (α_y) = lamina transverse coefficient of thermal expansion

STFRTM = stress free temperature (i.e., the curing temperature of the laminate)

2. Read (5,110) THICK (not inputted if ITYPE = 1)

110 Format (G10.4)

THICK = laminate thickness

3. Read (5,120) (J,PLYTHK (I), I = 1, NPLY) (not inputted if ITYPE = 0)

120 FORMAT (I3, G10.4)

J = ply number

PLYTHK (I) = ply thickness

4. Read (5,130) (J, ANGLE (I), TEM (I), I = 1, NPLY)

130 Format (I3, 2G10.4)

J = ply number

ANGLE (I) = ϕ_i = ply orientation angle

TEM (I) = T_i = ply temperature

5. Read (5,140) ZN1, ZN2, ZN3
140 Format (3610.4)
ZN1 = N_1 = normal force along laminate 1-axis
ZN2 = N_2 = normal force along laminate 2-axis
ZN3 = N_3 = shear force on laminate
6. Read (6,140) ZM1, ZM2, ZM3
ZM1 = M_1 = bending moment on laminate 1-face
ZM2 = M_2 = bending moment on laminate 2-face
ZM3 = M_3 = twisting moment on laminate
7. Read (5,170) X, XPRIME, Y, YPRIME, SS, FXYSTR
170 Format (6G10.4)
X = longitudinal tensile strength of lamina
XPRIME (X') = longitudinal compressive strength of lamina
Y = transverse tensile strength of lamina
YPRIME = (Y') = transverse compressive strength of lamina
SS (S) = lamina shear strength
FXSTR (F_{xy}^*) = 0.5 (in accordance with Eq. (59))

A sample output of a typical analysis is presented on the pages following this page. This is the problem of a cross-ply E-glass/epoxy laminate subjected to uniaxial loading, i.e.

(N_1, N_2, N_3) = (1,0,0). The cross-ply ratio, M, is 0.2. The curing temperature is 270°F, and the temperature of the laminate is 70°F.

NO. OF PLYS ABOVE MIDPLANE= 2 I- I TYPE= 1
 I TYPE=0 FOR UNIFORM PLY THICKNESSES
 I TYPE=1 FOR NON UNIFORM PLY THICKNESSES

LAMINA ELASTIC PROPERTIES W.R.T. LAMINA COORDINATES:

NUXY= .250 NUYX= .833D-01 EX=.7800D+07 EY=.2600D+07 G=.1250D+07
 ALPHAX= .3500D-05 ALPHAY= .1140D-04

THE STRESS FREE TEMPERATURE IS 270.0

LAMINATE CONSTRUCTION:

PLY	ANGLE	PLY THICKNESS	TEM
1	90.00	41.67	70.00
2	.0	.8333D-01	70.00

Z-LOCATION

0
 41.67
 41.75

INPLANE LOADS W.R.T. LAMINATE AXES:

ZN1= 1.000 ZN2= .0 ZN3= .0

MOMENTS W.R.T. LAMINATE AXES:

ZM1= .0 ZM2= .0 ZM3= .0

*** MULTIPLICATION OF THE ABOVE LOADS AND MOMENTS BY THE STRENGTH RATIO AT LAMINATE FAILURE GIVES THE FAILURE LOADS AND MOMENTS

STIFFNESS COEFFICIENTS IN NATURAL COORDINATES:

QXX= .757D+07 QYY= .266D+07 QXY= .664D+06 QSS= .125D+07

STRENGTHS W.R.T. LAMINA AXES:

X= .1500D+06 XPRIME= .1500D+06 Y= 4000. YPRIME= .2000D+05 SS= 6000. FXVSTR=-.5000

TSAI-WU STRENGTH PARAMETERS W.R.T. LAMINA AXES:

FXX .4444D-10 .0 FX .1250D-07 .2000D-03 FSS .2778D-07 -.3727D-09 FXY

ICYCLE= 1

PLY STIFFNESS COEFFICIENTS W.R.T. LAMINATE AXES:

PLY	QBAR11	QBAR22	QBAR12	QBAR66	QBAR16	QBAR26
1	.264D+07	.797D+07	.684D+06	.125D+07	.679D-02	.64D-01
2	.757D+07	.266D+07	.664D+06	.125D+07	.0	.0

APPLIED, THERMAL, AND TOTAL INPLANE FORCES:

ZN1= 1.000	ZNT1=-.5444D+06	TOTN1=-.5444D+06
ZN2= .0	ZNT2=-.5919D+06	TOTN2=-.5919D+06
ZN3= .0	ZNT3=-.7226D-02	TOTN3=-.7226D-02

THERMAL COEFFICIENTS OF EXPANSION W.R.T. LAMINATE AXES:

PLY	ALPHA1	ALPHA2	ALPHA6
1	.114D-04	.350D-05	.1055D-12
2	.350D-05	.114D-04	.0

A-MATRIX

.2226D+09	.5543D+08	.5657
.5543D+08	.6643D+09	5.342
.5657	5.342	.1044D+09

D-MATRIX

.1304D+12	.3221D+11	.327.4
.3221D+11	.3849D+12	3092.
.327.4	3092.	.6065D+11

PLATE THICKNESS= 83.50

A-INVERSE MATRIX

.4588D-08	-.3828D-09	-.5271D-17
-.3828D-09	.1537D-08	-.7661D-16
-.5271D-17	-.7661D-16	.9581D-08

D-INVERSE MATRIX

.7833D-11	-.6553D-12	-.8874D-20
-.6553D-12	.2653D-11	-.1317D-18
-.8874D-20	-.1317D-18	.1649D-10

STRESSES IN LAMINA AND LAMINATE COORDINATES BEFORE MULTIPLICATION WITH THE LOAD FACTOR:

PLY	STRS11	STRS12	STRS13	DEL1EM
1	.22.93	-.6.297	.5255D-07	-200.0
2	-.1147D+05	3148.	-.2627D-04	-200.0
PLY	STRSL1	STRSL2	STRSL3	
1	.1153D-01	-.4058D-05	.1318D-13	
2	.3629D-01	.2029D-02	-.6589D-11	
PLY	STRSX	STRSY	TAUXY	R1
1	-.6.297	22.93	-.4427D-06	.3334D+06
2	-.1147D+05	3148.	-.2627D-04	.4406D+06
PLY	STRSLX	STRSLY	TAULXY	R2
1	-.4058D-05	.1193D-01	-.1593D-09	-.1679D+07
2	.3629D-01	.2029D-02	-.6589D-11	-.8800D+07

CYCLE= 1 IFAIL= 1 LOAD FACTOR= .3334D+06

STRESSES AT IMPENDING PLY FAILURE AFTER MULTIPLICATION W. THE LOAD FACTOR:

PLY	STRESS	STRESSY	TAUXY
1	-7.650	4000.	-.5355D-04
2	.633.5	3825.	-.2847D-04

ICVCL= 2

PLY STIFFNESS COEFFICIENTS W.R.T. LAMINATE AXES:

PLY	QBAR11	QBAR22	QBAR12	QBAR66	QBAR16	QBAR26
2	.757D+07	.266D+07	.664D+06	.125D+07	.0	.0

APPLIED, THERMAL, AND TOTAL INPLANE FORCES:

ZN1=	1.000	ZNT1=-1182.	TOTN1=-1181.
ZN2=	.0	ZNT2=-1086.	TOTN2=-1086.
ZN3=	.0	ZNT3=.0	TOTN3=.0

THERMAL COEFFICIENTS OF EXPANSION W.R.T. LAMINATE AXES:

PLY	ALPHA1	ALPHA2	ALPHA6
2	.3500D-05	.1140D-04	.0

A-MATRIX

	.1328D+07	.1106D+06	.0	.2310D+10	.1925D+09	.0
	.11C6D+06	.4425D+06	.0	.1925D+09	.7698D+09	.0
	.0	.2083D+06	.0	.0	.3624D+09	.0

D-MATRIX

PLATE THICKNESS= 83.33

A-INVERSE MATRIX

	.7653D-06	-.1923D-06	.0	.4422D-09	-.1106D-09	.0
	-.1923D-06	.2308D-05	.0	-.1106D-09	.1327D-08	.0
	.0	.4800D-05	.0	.0	.2759D-08	.0

D-INVERSE MATRIX

STRESSES IN LAMINA AND LAMINATE COORDINATES BEFORE MULTIPLICATION WITH THE LOAD FACTOR

PLY	STRST1	STRST2	STRST3	DELTEM
2	-.2885D-09	-.2658D-09	.0	-200.0
PLY	STRSL1	STRSL2	STRSL3	
2	6.000	.0	.0	
PLY	STRSTX	STRSTY	TAUTXY	R1
2	-.2889D-09	-.2658D-09	.0	.2500D+05
PLY	STRSLX	STRSLY	TAULXY	R2
2	6.000	.0	.0	-.2500D+05

CYCLE= 2 IFAIL= 2 LOAD FACTOR= .2500D+05

STRESSES AT IMPENDING PLY FAILURE AFTER MULTIPLICATION W. THE LOAD FACTOR:

PLY	STRESX	STRESY	TAUXY
2	.1500D+06	-.2658D-09	.0

*** THE STRENGTH RATIO FOR THE LAMINATE IS .3334D+06

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